MATH 3631
Actuarial Mathematics II
Class Test 1 - 5:00-6:15 PM
Wednesday, 14 February 2018
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL Student ID: Suggested Solutions

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

An insurer sells a portfolio of 702 fully discrete whole life insurance policies with death benefit of 1000 to lives with independent future lifetimes, each with age x. You are given:

- The annual contract premium is 16 per policy.
- $i = 0.04$ $A_x = 0.306$ ${}^2A_x = 0.113$
- A table of α-th percentile, z_α, from the standard normal distribution:

$d = \frac{.04}{1.04}$

α	0.90	0.95	0.97	0.99
z_α	1.282	1.645	1.881	2.326

Calculate the probability of a gain from this portfolio of policies.

For one policy, $L_{0,i} = 1000v^{K+1} - 16\ddot{a}_{\overline{K+1|}} = \left(1000 + \frac{16}{d}\right)v^{K+1} - \frac{16}{d}$

$E[L_{0,i}] = 1000A_x - 16\ddot{a}_x = 1000A_x - 16\left(\frac{1-A_x}{d}\right) = 17.296$

$Var[L_{0,i}] = \left(1000 + \frac{16}{d}\right)^2\left[{}^2A_x - (A_x)^2\right] = 38825.9$

Let L = aggregate loss

$E[L] = 702(17.296) = 12141.79$

$Var[L] = 702(38825.9) = 27255785$

$\Pr[L<0] \cong \Pr\left[N < \underbrace{\frac{0 - 12141.79}{\sqrt{27255785}}}_{-2.326}\right] = 1 - \Pr[N \leq 2.326]$

$= 1 - (0.99) = \underline{\underline{0.01}}$

0.01 EV corrected 3/4/2018

Question No. 2:

For a special whole life insurance on (45), you are given:

- Death benefit, payable at the end of the year of death, consists of 250 plus the return of all premiums without interest.
- Annual net premium of 5 is payable at the beginning of each year.
- $A_{45} = 0.25$ $\ddot{a}_{45} = 21.7$

Calculate $(IA)_{45}$.

$$APV(FP_0) = APV(FB_0)$$

$$5\ddot{a}_{45} = 250 A_{45} + 5(IA)_{45}$$

$$(IA)_{45} = \ddot{a}_{45} - 50 A_{45}$$

$$= 21.7 - 50(.25)$$

$$= \underline{\underline{9.2}}$$

Question No. 3:

For a whole life insurance issued to (40), you are given:

- The death benefit is 100, payable at the end of the year of death.
- There is only a single gross premium of 14, payable at policy issue. single
- Initial expenses are 4% of the single gross premium.
- Additional expenses of 0.05 incurred at the beginning of each year, including the first policy year.
- Mortality follows the `Illustrative Life Table`.
- $\delta = 0.05$
- L_0 is the loss at issue for this policy.

$d = 1 - v = 1 - e^{-.05}$

$\frac{1 - v^{K+1}}{d}$

Calculate $\Pr[L_0 > 15]$.

$$L_0 = 100v^{K+1} + .04(14) + .05\,\ddot{a}_{\overline{K+1}|} - 14$$

$$= \left(100 - \frac{.05}{d}\right)v^{K+1} + \left(\frac{.05}{d} - 13.44\right)$$

$$= 98.97479\,v^{K+1} - 12.41479$$

$$\Pr[L_0 > 15] = \Pr\Big[v^{K+1} > \underbrace{\frac{15 + 12.41479}{98.97479}}_{0.2769876}\Big]$$

$$= \Pr\Big[K < \underbrace{\frac{\log(0.2769876)}{-\delta} - 1}_{24.67565}\Big]$$

$$= \Pr[K \leq 24] = {}_{25}q_{40}$$

$$= 1 - \frac{l_{65}}{l_{40}} = \underline{\underline{0.1910416}}$$

7533964

9313166

Question No. 4:

For a fully discrete whole life insurance of 100 on (40), you are given:

- First year expenses are 25% of the gross premium.
- Renewal expenses are 5% of the gross premium.
- Expenses are incurred at the beginning of the policy year.
- Gross premium is calculated according to the equivalence principle.
- Mortality follows the `Illustrative Life Table` with $i = 0.06$.

Calculate the gross premium reserve at the end of the second year.

$$G\ddot{a}_{40} = 100A_{40} + 0.20G + 0.05G\ddot{a}_{40}$$

$$G = \frac{100\overbrace{A_{40}}^{.16132}}{.95\underbrace{\ddot{a}_{40}}_{14.8166} - 0.20} = 1.162602$$

Use recursive formula

$${}_0V = 0$$

$${}_1V = \frac{(0 + G(.75))(1.06) - 100\overbrace{q_{40}}^{2.78/1000}}{1 - q_{40}} = 0.6480703$$

$${}_2V = \frac{({}_1V + G(.95))(1.06) - 100\overbrace{q_{41}}^{2.98/1000}}{1 - q_{41}} = \underline{\underline{1.564357}}$$

alternatively: ${}_2V = 100A_{42} + .05G\ddot{a}_{42} - G\ddot{a}_{42}$

$$= 100A_{42} - .95G\ddot{a}_{42} = \underline{\underline{1.564827}}$$

$$= 100(.17636) - .95G(14.5510)$$

Question No. 5:

For a fully discrete 10-year term insurance policy of 10 on (50), you are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$

Calculate the net premium reserve at the end of 9 years.

$$P = 10 \frac{A^{1}_{50:\overline{10}|}}{\ddot{a}_{50:\overline{10}|}} = 10 \frac{A_{50} - {}_{10}E_{50}\, A_{60}}{\ddot{a}_{50} - {}_{10}E_{50}\, \ddot{a}_{60}} = 10 \underbrace{\frac{.24905 - .51081(.36913)}{13.2668 - .51081(11.1454)}}_{= 0.007987557}$$

5

$${}_9V = APV(FB_9) - APV(FP_9)$$

$$= 10 \cdot v \cdot q_{59} - P$$

5

$$= \underbrace{10 \frac{1}{1.06} \frac{12.62}{1000} - 0.007987557}$$

$$= \cancel{0.111069}$$

0.03918104

Question No. 6:

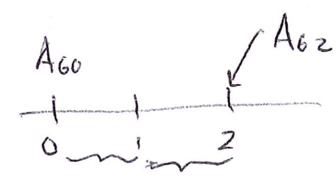
For a fully discrete whole life insurance of 1 on (60), you are given:

- $q_{60} = 0.003 \qquad q_{61} = 0.004$
- $i = 0.05$
- $A_{60} = 0.30$
- ${}^2A_{60} = 0.10$
- L_t is the insurer's prospective loss at time t for this policy.

Calculate $\mathrm{Var}(L_2)$.

$$P = \frac{A_{60}}{\ddot{a}_{60}} = \frac{A_{60}}{\left(\frac{1-A_{60}}{d}\right)} = \frac{0.30\left(\frac{.05}{1.05}\right)}{1-0.30} = 0.0204816$$

$$\mathrm{Var}[L_2] = \left(1+\frac{P}{d}\right)^2\left[{}^2A_{62} - (A_{62})^2\right]$$



$$A_{60} = v\,q_{60} + v^2 p_{60}\,q_{61} + v^2 p_{60}\,p_{61}\,A_{62}$$

and solve for $A_{62} = \dfrac{A_{60} - v\,q_{60} - v^2 p_{60}\,q_{61}}{v^2 p_{60}\,p_{61}}$

$$= 0.3258893$$

$${}^2A_{60} = v^2 q_{60} + v^4 p_{60}\,q_{61} + v^4 p_{60}\,p_{61}\,{}^2A_{62}$$

and solve for ${}^2A_{62} = \dfrac{{}^2A_{60} - v^2 q_{60} - v^4 p_{60}\,q_{61}}{v^4 p_{60}\,p_{61}}$

$$= 0.1150592$$

$$\mathrm{Var}[L_2] = \left(1+\frac{0.0204816}{.05/1.05}\right)^2\left[0.1150592 - (0.3258893)^2\right] = \underline{\underline{0.01807207}}$$

Question No. 7:

For a fully discrete whole life insurance of 1,000,000 on (45), you are given:

- First year expenses are 20% of the gross premium plus 3000.
- Renewal expenses are 2% of the gross premium plus 300.
- All expenses are incurred at the beginning of the policy year.
- Gross premiums are calculated using the equivalence principle.
- Mortality follows the `Illustrative Life Table` with $i = 0.06$.

$A_{45} = 0.20120$

$\ddot{a}_{45} = 14.1121$

Calculate the gross premium reserve at the end of the first policy year. ✓

$$G\ddot{a}_{45} = 1000000 A_{45} + 2700 + 0.18G + 300\ddot{a}_{45} + .02G\ddot{a}_{45}$$

$$G = \frac{1000000 A_{45} + 2700 + 300\ddot{a}_{45}}{.98\ddot{a}_{45} - 0.18}$$

$$= 15248.04$$

Since only the first year, use recursive formula

$q_{45} = \frac{4.00}{1000}$

${}_0V = 0$

$${}_1V = \frac{(G(.80) - 3000)(1.06) - 1000000\, q_{45}}{1 - q_{45}}$$

$$= \underline{\underline{5773.435}}$$

Question No. 8:

For a life insurance policy issued to (x), you are given:

- Death benefit of 25,000 is payable at the end of the year of death.
- The annual net premium in year 16, payable at the beginning of the year, is 654.57.
- Deaths are assumed to be uniformly distributed over integral ages.
- $i = 0.05$ $\quad {}_{15}V = 9{,}227.79$ $\quad {}_{15.5}V = 9{,}889.50$

Calculate q_{x+15}.

no expenses

use mid-year formula

$$_{15.5}V = \frac{({}_{15}V + P)(1+i)^{0.5} - 25000\ {}_{0.5}q_{x+15}\ v^{0.5}}{1 - {}_{0.5}q_{x+15}}$$

By UDD, ${}_{0.5}q_{x+15} = 0.5\, q_{x+15}$. Solving for q_{x+15} we get

$$q_{x+15} = \frac{({}_{15}V + P)(1+i)^{0.5} - {}_{15.5}V}{(25000 v^{0.5} - {}_{15.5}V)(0.5)}$$

$$= \frac{(9227.79 + 654.57)(1.05)^{0.5} - 9889.50}{\left(25000\left(\frac{1}{1.05}\right)^{0.5} - 9889.50\right)(0.5)}$$

$$= \frac{236.9056}{7254.001} = \underline{\underline{0.0326586 2}}$$

Question No. 9:

For a 5-year endowment insurance on (60), you are given:

- The death benefit, payable at the end of the year of death, is equal to 1000 plus the benefit reserve.
- The endowment benefit is 4000.
- Level premiums, P, are payable annually at the beginning of each year.
- $q_{60+k} = 0.02$, for $k = 0, 1, 2, \ldots$
- $i = 0.05$

Calculate P.

Use recursive formula with ${}_0V = 0$ and ${}_5V = 4000$

$$ {}_1V = P(1.05) - (1000 + \cancel{{}_1V} - \cancel{{}_1V})\, \underbrace{q_{60}}_{.02} $$

$$ = P(1.05) - 1000(.02) $$

$$ {}_2V = ({}_1V + P)(1.05) - (1000 + \cancel{{}_2V} - \cancel{{}_2V})(.02) $$

$$ = P[(1.05)^2 + 1.05] - 1000(.02)(1.05 + 1) $$

$$ \vdots $$

$$ {}_5V = P\Big[\overbrace{\underbrace{1.05^5 + 1.05^4 + \ldots + 1.05}_{\frac{1.05(1.05^5-1)}{.05}}}^{5.801913}\Big] - 20\Big(\overbrace{\underbrace{1.05^4 + 1.05^3 + \ldots + 1}_{\frac{1.05^5-1}{.05}}}^{110.5126}\Big) $$

and equate to 4000

$$ P = \frac{4000 + 110.5126}{5.801913} = \underline{\underline{708.4754}} $$

Question No. 10:

For a fully discrete whole life insurance policy of 1 on (35), you are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$
- Expenses consist of 5% of annual gross premium, payable at the beginning of each year.
- Both the annual net premium and the annual gross premium are determined according to the equivalence principle.
- ${}_tV^n$ and ${}_tV^g$ denote the net and gross premium reserves at time t, respectively.

Calculate ${}_{10}V^n - {}_{10}V^g$.

Expenses are flat. No extra expense in the first year

Therefore, difference is zero!

$${}_{10}V^n - {}_{10}V^g = 0$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK