MATH 3631
Actuarial Mathematics II
Class Test 1 - 5:00-6:15 PM
Wednesday, 15 February 2017
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: _____   Student ID: _____

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.

- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.

- Please write legibly.

- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

An insurance company sells N fully discrete whole life insurance policies with death benefit of 200, each with the same age x. You are given:

- The annual contract premium is 5.50 per policy.

- $i = 0.05$

- $A_x = 0.35$

- $^2A_x = 0.17$

- All policies have independent future lifetimes.

- The 95th percentile on a standard normal distribution is 1.645.

Determine the smallest N so that the company has at least a 95% probability of a gain from this portfolio of policies.

Question No. 2:

You are given the following information about a special fully discrete 2-payment, 2-year endowment life insurance on (45):

- The death benefit is 100 plus a return of all premiums accumulated with interest at an annual effective rate of 4%.

- The endowment benefit is 200.

- Mortality is based on: $q_{45} = 0.01$ $q_{46} = 0.02$

- $i = 0.10$

- Level premiums are calculated based on the equivalence principle.

Calculate the net annual premium for this insurance.

Question No. 3:

For a fully discrete whole life insurance issued to (40), you are given:

- The death benefit is 100.

- Mortality follows the Illustrative Life Table.

- $i = 0.06$

Calculate the net premium reserve at the end of 10 years.

Question No. 4:

For a fully discrete whole life insurance of 100 on (40), you are given:

- First year expenses are 25% of the gross premium.

- Renewal expenses are 5% of the gross premium.

- Expenses are incurred at the beginning of the policy year.

- Gross premium is calculated according to the equivalence principle.

- Mortality follows the `Illustrative Life Table` with $i = 0.06$.

Calculate the gross premium reserve at the end of the second year.

Question No. 5:

For a fully discrete whole life insurance of 1 on (45), you are given:

- $q_{50} = 0.003$

- $A_{51} = 0.20$

- $^2A_{51} = 0.07$

- $i = 0.05$

- L_k is the insurer's prospective loss at time k for this policy.

Calculate $\dfrac{\text{Var}(L_5)}{\text{Var}(L_6)}$.

Question No. 6:

For a fully discrete whole life insurance of 1000 on (x), you are given:

- The gross premium reserve at duration 9 is 109 and at duration 10 is 124.

- $q_{x+9} = 0.003$

- $i = 0.05$

- Renewal expenses at the start of each year are 1 plus 2% of the gross premium.

- There are no associated expenses at death.

Calculate the annual gross premium.

Question No. 7:

For a 10-year endowment insurance on (50), you are given:

- The death benefit, payable at the end of the year of death, is equal to 100 plus the benefit reserve.

- The endowment benefit is 500, payable at the end of 10 years if alive.

- Level premiums, π, are payable annually at the beginning of each year.

- $q_{50+k} = 0.01$, for $k = 0, 1, 2, \ldots$

- $i = 4\%$

Calculate π.

Question No. 8:

For a life insurance policy issued to (50), you are given:

- Death benefit of 1 is payable at the end of the year of death.

- The benefit premium in year 11, payable at the beginning of the year, is 0.045.

- There are no expenses for this policy.

- The policy is still active after 10 years.

- Deaths are assumed to be uniformly distributed over integral ages.

- $q_{60} = 0.080$

- $i = 0.05$

- $_{10}V = 0.325$

Calculate $_{10.75}V$.

Question No. 9:

An insurer issued 4,000 fully discrete whole life insurance policies to lives all exactly age 50 on January 1, 2006. Each policy issued has a death benefit of 100,000 with an annual gross premium of 2,600.

You are given:

- The following values in Year 2015:

	anticipated	actual
Expenses as a percent of premium	0.05	0.06
Annual effective rate of interest	0.02	0.05
q_{59}	0.0085	0.0090

- The gross premium reserves per policy at the end of Year 2014 and Year 2015, respectively, are:
$$_9V = 17,033 \quad \text{and} \quad _{10}V = 19,206$$

- A total of 3,851 remain in force at the beginning of Year 2015.

- Gains and losses are calculated in the following order: expenses then interest then mortality.

Calculate the gain (or loss) from each source (expensese, interest, mortality) for this portfolio of policies in Year 2015.

Question No. 10:

XYZ Life Insurance Company issues 5,000 fully discrete whole life insurance policies of 10,000 to lives each age 50, with independent future lifetimes. You are given:

- The annual gross premium is 220 per policy.

- Each policy is assumed to incur an expense of 30 at the beginning of each year.

- Gross premiums and reserves are calculated using $q_{53} = 0.0068$ and $i = 0.05$.

- At the end of the third policy year:

    i. The gross premium reserve per policy is 505.
    ii. There are 4,900 policies in force.

- During the fourth policy year:

    i. The actual expense incurred per policy was 28.
    ii. There were a total of 40 actual deaths.
    iii. The actual interest rate earned was 6.5%.

Calculate the total gain or loss for the fourth policy year.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK