MATH 3631
Actuarial Mathematics II
Class Test 1 - 5:00-6:15 PM
Wednesday, 15 February 2017
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL

Student ID: Suggested Solutions

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

An insurance company sells N fully discrete whole life insurance policies with death benefit of 200, each with the same age x. You are given:

- The annual contract premium is 5.50 per policy.
- $i = 0.05$
- $A_x = 0.35$
- ${}^2A_x = 0.17$
- All policies have independent future lifetimes.
- The 95th percentile on a standard normal distribution is 1.645.

Determine the smallest N so that the company has at least a 95% probability of a gain from this portfolio of policies.

Let the loss per policy be $L_{0,i} = 200 v^{K+1} - 5.5\ddot{a}_{\overline{K+1|}}$

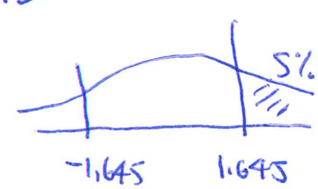
$$= \left(200 + \frac{5.5}{d}\right) v^{K+1} - 5.5/d$$

$$E[L_{0,i}] = \left(200 + \frac{5.5}{d}\right)A_x - \frac{5.5}{d} = \left(200 + \frac{5.5}{.05/1.05}\right)(.35) - \frac{5.5}{.05/1.05} = -5.075$$

$$Var[L_{0,i}] = \left(200 + \frac{5.5}{d}\right)^2\left({}^2A_x - (A_x)^2\right) = \left(200 + \frac{5.5}{.05/1.05}\right)\left(.17 - .35^2\right) = 4728.162$$

$$L_{agg} = \sum_{i=1}^{N} L_{0,i} \Rightarrow E[L_{agg}] = -5.075N \text{ and } Var[L_{agg}] = 4728.162N$$

$$P_r[L_{agg} < 0] \cancel{\geq} .95 \iff P_r\left[Z < \frac{-5.075N}{\sqrt{4728.126N}}\right] \cancel{\geq} .95$$



$$\frac{5.075\sqrt{N}}{\sqrt{4728.126}} \geq 1.645$$

$$\iff \sqrt{N} \geq \frac{1.645\sqrt{4728.126}}{5.075} = 22.28825$$

$$N \geq 496.7662$$

$\underline{\underline{N = 497 \text{ policies}}}$

Question No. 2:

You are given the following information about a special fully discrete 2-payment, 2-year endowment life insurance on (45):

- The death benefit is 100 plus a return of all premiums accumulated with interest at an annual effective rate of 4%.
- The endowment benefit is 200.
- Mortality is based on: $q_{45} = 0.01$ $q_{46} = 0.02$
- $i = 0.10$
- Level premiums are calculated based on the equivalence principle.

Calculate the net annual premium for this insurance.

$$v = \frac{1}{1.10}$$

$$APV(FP_0) = P(1 + v(.99))$$

$$APV(FB_0) = \{100 + P(1.04)\} v(.01) + [100 + P(1.04)^2 + P(1.04)] \times v^2(.99)(.02) + 200v^2(.99)(.98)$$

Solving for P, we get

$$P = \frac{100[v(.01) + v^2(.99)(.02)] + 200v^2(.99)(.98)}{1 + v(.99) - 1.04v(.01) - [1.04^2 + 1.04]v^2(.99)(.02)}$$

$$= \frac{162.9091}{1.855828} = \underline{\underline{87.78241}}$$

Question No. 3:

For a fully discrete whole life insurance issued to (40), you are given:

- The death benefit is 100.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$

Calculate the net premium reserve at the end of 10 years.

$$P = 100 \frac{A_{40}}{\ddot{a}_{40}} = 100 \frac{.16132}{14.8166} = 1.088779$$

$$_{10}V = APV(FB_{10}) - APV(FP_{10})$$
$$= 100 A_{50} - P \ddot{a}_{50} = 100(.24905) - P(13.2668)$$
$$= 10.46039$$

OR use

$$_{10}V = 100\left(1 - \frac{\ddot{a}_{50}}{\ddot{a}_{40}}\right) = 100\left(1 - \frac{13.2668}{14.8166}\right)$$
$$= 10.45989$$

The difference is on rounding!

Question No. 4:

For a fully discrete whole life insurance of 100 on (40), you are given:

- First year expenses are 25% of the gross premium.
- Renewal expenses are 5% of the gross premium.
- Expenses are incurred at the beginning of the policy year.
- Gross premium is calculated according to the equivalence principle.
- Mortality follows the `Illustrative Life Table` with $i = 0.06$.

$A_{40} = .16132$

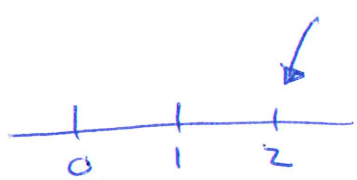
$\ddot{a}_{40} = 14.8166$

Calculate the gross premium reserve at the end of the second year.

Let G = gross annual premium

$$G\ddot{a}_{40} = 100A_{40} + .20G + .05G\ddot{a}_{40}$$

$$G = \frac{100A_{40}}{.95\ddot{a}_{40} - .20} = 1.162602$$



Use first principle or recursion

$${}_2V = APV(FB_2) + APV(FE_2) - APV(FP_2)$$

$$= 100A_{42} + .05G\ddot{a}_{42} - G\ddot{a}_{42}$$

$$= 100(.17636) - .95(1.162602)(14.5510)$$

$$= \underline{\underline{1.564827}}$$

Question No. 5:

For a fully discrete whole life insurance of 1 on (45), you are given:

- $q_{50} = 0.003$
- $A_{51} = 0.20$
- ${}^2A_{51} = 0.07$
- $i = 0.05$
- L_k is the insurer's prospective loss at time k for this policy.

Calculate $\dfrac{\text{Var}(L_5)}{\text{Var}(L_6)}$.

$$V = \frac{1}{1.05}$$

$$\frac{\text{Var}(L_5)}{\text{Var}(L_6)} = \frac{\left(1+\cancel{P_{45}}/d\right)^2\left({}^2A_{50} - A_{50}^2\right)}{\left(1+\cancel{P_{45}}/d\right)^2\left({}^2A_{51} - A_{51}^2\right)}$$

$$A_{50} = v\,q_{50} + v\,p_{50}A_{51} = v(.003) + v(.997)(.20) = 0.1927619$$

$${}^2A_{50} = v^2 q_{50} + v^2 p_{50}\,{}^2A_{51} = v^2(.003) + v^2(.997)(.07) = 0.06602268$$

$$= \frac{0.06602268 - (0.1927619)^2}{.07 - (.20)^2}$$

$$= \frac{.02886552}{.03} = \underline{\underline{0.9621841}}$$

Question No. 6:

For a fully discrete whole life insurance of 1000 on (x), you are given:

- The gross premium reserve at duration 9 is 109 and at duration 10 is 124.
- $q_{x+9} = 0.003$
- $i = 0.05$
- Renewal expenses at the start of each year are 1 plus 2% of the gross premium.
- There are no associated expenses at death.

Calculate the annual gross premium.

$$ {}_{10}V = \frac{({}_9V + G - 1 - .02G)(1.05) - 1000(.003)}{1 - .003} $$

$$ \underbrace{.997(124) + 1000(.003) - 1.05(109) + 1.05}_{13.228} = 0.98G(1.05) $$

Solving for G, we get

$$ G = \frac{13.228}{1.05 * 0.98} = 12.8552 $$

Question No. 7:

For a 10-year endowment insurance on (50), you are given:

- The death benefit, payable at the end of the year of death, is equal to 100 plus the benefit reserve.
- The endowment benefit is 500, payable at the end of 10 years if alive.
- Level premiums, π, are payable annually at the beginning of each year.
- $q_{50+k} = 0.01$, for $k = 0, 1, 2, \ldots$
- $i = 4\%$

Calculate π.

Use recursive formula

$$_0V = 0$$

$$_1V = \pi(1.04) - (100 + \cancel{_1V} - \cancel{_1V})(.01) = 1.04\pi - 100(.01)$$

$$_2V = (1.04\pi - 100(.01) + \pi)(1.04) - (100 + \cancel{_2V} - \cancel{_2V}).01$$

$$= \pi(1.04^2 + 1.04) - 100(.01 + .01(1.04))$$

$\vdots$

$$_{10}V = \pi\underbrace{\left[1.04^{10} + \ldots + 1.04\right]}_{1.04\left(\frac{1.04^{10}-1}{.04}\right)} - 100(.01)\underbrace{\left[1 + 1.04 + \ldots + 1.04^9\right]}_{\frac{1.04^{10}-1}{.04}}$$

$= 500$, the endowment

$$\pi = \frac{500 + \frac{1.04^{10}-1}{.04}}{1.04\left(\frac{1.04^{10}-1}{.04}\right)} = \underline{\underline{41.00526}}$$

Question No. 8:

For a life insurance policy issued to (50), you are given:

- Death benefit of 1 is payable at the end of the year of death. $P = .045$
- The benefit premium in year 11, payable at the beginning of the year, is 0.045.
- There are no expenses for this policy.
- The policy is still active after 10 years.
- Deaths are assumed to be uniformly distributed over integral ages.
- $q_{60} = 0.080$
- $i = 0.05$
- ${}_{10}V = 0.325$

10   10.75   11

Calculate ${}_{10.75}V$.

$${}_{10.75}V = \frac{({}_{10}V + P)(1+i)^{.75} - {}_{.75}q_{60} * v^{.25}}{1 - {}_{.75}q_{60}}$$

$$= \frac{(0.325 + 0.045)(1.05)^{.75} - .75(.080)(1.05)^{-.25}}{1 - .75(.080)}$$

$$= \frac{0.3245174}{0.94}$$

$$= \underline{\underline{0.3452313}}$$

Question No. 9:

An insurer issued 4,000 fully discrete whole life insurance policies to lives all exactly age 50 on January 1, 2006. Each policy issued has a death benefit of 100,000 with an annual gross premium of 2,600.

You are given:

- The following values in Year 2015:

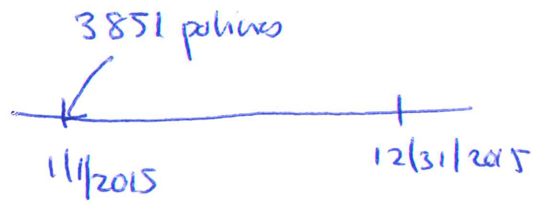
	anticipated	actual
Expenses as a percent of premium	0.05	0.06
Annual effective rate of interest	0.02	0.05
q_{59}	0.0085	0.0090

- The gross premium reserves per policy at the end of Year 2014 and Year 2015, respectively, are:

$$_{9}V = 17,033 \quad \text{and} \quad _{10}V = 19,206$$

- A total of 3,851 remain in force at the beginning of Year 2015.
- Gains and losses are calculated in the following order: expenses then interest then mortality.

Calculate the gain (or loss) from each source (expensese, interest, mortality) for this portfolio of policies in Year 2015.



expenses: $3851[2600(.05-.06)](1.02) = -102,128.5$ loss

interest: $3851[17033 + 2600(.94)](.05-.02) = 2,250,178$ gain

mortality: $3851(100,000 - 19206)(.0085-.0090) = -155,568.8$ loss

total gain/loss is the sum of these 3 sources: 1,992,480 gain

Question No. 10:

XYZ Life Insurance Company issues 5,000 fully discrete whole life insurance policies of 10,000 to lives each age 50, with independent future lifetimes. You are given:

- The annual gross premium is 220 per policy.
- Each policy is assumed to incur an expense of 30 at the beginning of each year.
- Gross premiums and reserves are calculated using $q_{53} = 0.0068$ and $i = 0.05$.
- At the end of the third policy year:
  i. The gross premium reserve per policy is 505.
  ii. There are 4,900 policies in force.
- During the fourth policy year:
  i. The actual expense incurred per policy was 28.
  ii. There were a total of 40 actual deaths.
  iii. The actual interest rate earned was 6.5%.

Calculate the total gain or loss for the fourth policy year.

$$ {}_4V^E = \frac{(505+220-30)(1.05) - 10000\,\overset{.0068}{q_{50}}}{1-q_{50}} = 666.2807 $$

The reserves based on actual experience is

$$ {}_4V^A = (505+220-28)(1.065) - (10000-666.2807)\left(\frac{40}{4900}\right) $$

$$ = 666.1114 $$

The gain/loss for the 4th policy year is -829.702

$$ 4900(666.1114 - 666.2807) = $$

EV

2/22/2017

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK