MATH 3631
Actuarial Mathematics II
Class Test 1 - 3:35-4:50 PM
Wednesday, 17 February 2016
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL

Student ID: Suggested Solutions

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

For a life (x) with standard mortality, you are given:

- $\ddot{a}_x = 10.13$ when calculated at $\delta = 3.0\%$
- $\ddot{a}_x = 9.17$ when calculated at $\delta = 4.5\%$

A fully discrete whole life insurance policy with death benefit of $10,000 is issued to a life (x) considered substandard risk with force of mortality:

$$\mu^s_{x+t} = \mu_{x+t} + 0.015,$$

where μ_{x+t} is the force of mortality for a standard risk.

Calculate the net annual premium for this substandard life if $\delta = 3.0\%$.

$${}_tp_x^s = e^{-\int_0^t (\mu_{x+s} + .015)ds} = {}_tp_x \, e^{-.015t}$$

$$\ddot{a}_x^s = \sum_{k=0}^{\infty} v^k \, {}_kp_x^s = \sum_{k=0}^{\infty} e^{-.03k} \, {}_kp_x \, e^{-.015k}$$

$$= \sum_{k=0}^{\infty} e^{-.045k} \, {}_kp_x = \ddot{a}_x \text{ @ } \delta = 4.5\%$$

$$= 9.17$$

$$A_x^s = 1 - d\,\ddot{a}_x = 1 - (1 - e^{-.03})(9.17) = 0.7289855$$

$$P^s = 10{,}000 \frac{A_x^s}{\ddot{a}_x^s} = \frac{10{,}000\,(.7289855)}{9.17} = \underline{\underline{794.9679}}$$

Question No. 2:

For a special fully discrete 20-year endowment insurance on (45), you are given:

- The death benefit is \$100 plus the return of all net annual premiums accumulated with interest at 4% to the end of the year of death.
- The endowment benefit is 200.
- $i = 4\%$
- $A^{\;\;1}_{45:\overline{20}|} = 0.3692$
- ${}_{20}p_{45} = 0.4312$
- Premiums are calculated based on the actuarial equivalence principle.

Calculate the net annual premium for this policy.

$$\text{Loss at issue} = L_0 = \begin{cases} 100v^{K+1} + \overbrace{P\ddot{s}_{\overline{K+1}|}v^{K+1} - P\ddot{a}_{\overline{K+1}|}}^{=0 \text{ since accumulation is also } i = 4\%}, & \text{for } K < 20 \\ 200v^{20} - P\ddot{a}_{\overline{20}|}, & \text{for } K \geq 20 \end{cases}$$

$$= 100v^{K+1} I(K < 20) + \left(200v^{20} - P\ddot{a}_{\overline{20}|}\right) I(K \geq 20)$$

$$E[L_0] = 100 A^{\;\;1}_{45:\overline{20}|} + \left(200v^{20} - P\ddot{a}_{\overline{20}|}\right) {}_{20}p_{45} = 0$$

Solving for P, we set

$v = 1/1.04$

$$P = \frac{100 A^{\;\;1}_{45:\overline{20}|} + 200v^{20}\, {}_{20}p_{45}}{\ddot{a}_{\overline{20}|}\, {}_{20}p_{45}}$$

$$\ddot{a}_{\overline{20}|} = \frac{1 - v^{20}}{1 - v} = 14.13394$$

$$= \frac{76.27881}{6.094555} = \underline{\underline{12.5159}}$$

Question No. 3:

For a 10-year term life insurance policy issued to (50), you are given:

- The death benefit of $100 is payable at the end of the year of death.
- A level premium is paid at the beginning of each year during the term of the policy.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$
- Net premium is calculated according to the actuarial equivalence principle.

Calculate the net premium reserve at the end of year 5.

$$P = 100\,\frac{A^{1}_{50:\overline{10}|}}{\ddot{a}_{50:\overline{10}|}} = \frac{100\left(\overset{.124905}{A_{50}} - {}_{10}\overset{.51081}{E_{50}}\,\overset{.36913}{A_{60}}\right)}{\left(\underset{13.2668}{\ddot{a}_{50}} - {}_{10}\underset{.51081}{E_{50}}\,\underset{11.1454}{\ddot{a}_{60}}\right)} = 100\left(\frac{.0604947}{7.573618}\right) = 0.7987557$$

$${}_5V = 100\,A^{1}_{55:\overline{5}|} - P\,\ddot{a}_{55:\overline{5}|}$$

$$= 100\left(\underset{.30514}{A_{55}} - {}_5\underset{.70810}{E_{55}}\,\underset{.36913}{A_{60}}\right) - P\left(\underset{12.2758}{\ddot{a}_{55}} - {}_5\overset{.70810}{E_{55}}\,\underset{11.1454}{\ddot{a}_{60}}\right)$$

$$= 100\,(.04375905) - P\,(4.383742)$$

$$= \underline{\underline{0.8743657}}$$

Question No. 4:

For a fully discrete whole life insurance policy of \$10,000 issued to (45), you are given:

- The only expense, incurred at policy issue, is \$100.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$
- Gross premium is determined according to the actuarial equivalence principle.

Calculate the gross premium reserve at the end of year 10.

$$G\ddot{a}_{45} = 10000 A_{45} + 100 \Rightarrow G = \frac{10000(.20120) + 100}{14.1121}$$

$$= 149.6588$$

$$_{10}V = 10000 A_{55} - G\ddot{a}_{55}$$

$$= 10000(.30514) - (149.6588)(12.2758)$$

$$= \underline{\underline{1214.218}}$$

Question No. 5:

For a fully discrete whole life insurance of $100,000 on (40), you are given:

- The gross premium reserve at time 10 is $16,025 and at time 11 is $17,830.
- The gross annual premium is $2,660.
- $i = 0.04$
- Renewal expenses, incurred at the beginning of each year, consist of 50 plus 3% of gross annual premium.
- There is an expense of $250 incurred at time death benefit is paid.

Calculate the probability that a 50-year-old will survive one year.

Using recursive formula, ${}_{11}V = ({}_{10}V + G*.97-50)(1.04)-(100{,}250 - {}_{11}V)\,q_{50}$

Solving for q_{50}, we get

$$q_{50} = \frac{(16025 + 2660(.97)-50)(1.04) - 17830}{100{,}250 - 17830}$$

$$= \frac{1467.408}{82420} = .01780403$$

$$p_{50} = 1 - q_{50} = \underline{\underline{0.982196}}$$

Question No. 6:

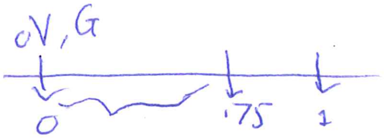
For a fully discrete whole life insurance of $1,000 on (35), you are given:

- First year expenses are 25% of the gross premium plus 75.
- Renewal expenses are 5% of the gross premium plus 25.
- All expenses are incurred at the beginning of the policy year.
- Gross premiums are calculated using the actuarial equivalence principle.
- Deaths are uniformly distributed over each year of age.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$

Calculate the gross premium reserve at the end of 0.75 years.

$$G\ddot{a}_{35} = 1000A_{35} + .20G + .05G\ddot{a}_{35} + 50 + 25\ddot{a}_{35}$$

$$G = \frac{1000A_{35} + 50 + 25\ddot{a}_{35}}{.95\ddot{a}_{35} - .20} = \frac{563.535}{14.42297} = 39.07205$$



$$ {}_{.75}V = \frac{(0 + G(.75) - 75)(1.06)^{.75} - 1000v^{.25}\,\overset{2.01/1000}{{}_{.75}q_{35}}}{1 - {}_{.75}q_{35} - 2.01/1000}$$

$$= \frac{-49.22293}{.9984925} = \underline{\underline{-.49.29725}}$$

Question No. 7:

For a special single premium 10-year endowment insurance on (55), you are given:

- The death benefit, payable at the end of the year of death, is equal to $2,000 plus the benefit reserve.
- The endowment benefit, payable at the end of 10 years if alive, is $5,000.
- $q_{55+k} = 0.01$, for $k = 0, 1, 2, \ldots$
- $i = 0.05$

Calculate the single benefit premium for this policy.

Let P = single benefit premium

$${}_1V = ({}_0V + P)(1.05) - (2000 + \cancel{{}_1V} - \cancel{{}_1V})\, \overset{.01 \nearrow}{q_{55}} = P(1.05) - 2000(.01)$$

$${}_2V = \left(P(1.05) - 2000(.01)\right)(1.05) - 2000(.01)$$

$$= P(1.05)^2 - 2000(.01)(1.05 + 1)$$

$$\vdots$$

$${}_{10}V = P(1.05)^{10} - 2000(.01)\left[\underbrace{1.05^9 + 1.05^8 + \ldots + 1}_{\frac{1.05^{10} - 1}{.05}}\right] = 5{,}000$$

Solving for P, we get

$$P = \frac{5000 + 20\left[\frac{1.05^{10} - 1}{.05}\right]}{(1.05)^{10}} = \frac{5251.558}{1.628895}$$

$$= \underline{\underline{3224.001}}$$

Question No. 8:

Suppose you are interviewing for an actuarial position at a major life insurance company. One of your interviewers is a qualified actuary with an FSA. She asked you about what you have learned from your class in 'Actuarial Mathematics' about reserves. Specifically, she asked 'why do you think every actuary should know about reserves and their calculations?'

How would you respond? Bullet points are acceptable. No formulas, and only words, please. Just like the song titled 'The Word' by the Beatles says "Say the word and you'll be free, say the word and be like me, say the word I'm thinking of".

- Reserves are needed to cover future obligations.
- Reserves are the biggest item on the liability side of the balance sheet and they also affect the income statement because any changes in reserves are considered expenses.
- Actuary is responsible for making sure calculations are done right and that enough company assets are there to back them up.

Question No. 9:

$$P = 100\,\frac{A_{50}}{\ddot{a}_{50}} = 100\,\frac{(1-d\ddot{a}_{50})}{\ddot{a}_{50}} = 5.244755$$

For a fully discrete whole life insurance of $100 issued to (50), you are given:

- $i = 0.04$
- $\ddot{a}_{50} = 11.0$ and $\ddot{a}_{60} = 8.0$
- ${}_{8}p_{\underset{60}{\cancel{50}}} = 0.750$ ${}_{9}p_{\underset{60}{\cancel{50}}} = 0.714$ ${}_{10}p_{\underset{60}{\cancel{50}}} = 0.676$ ${}_{11}p_{\underset{60}{\cancel{50}}} = 0.638$
- L_{10} denotes the prospective loss random variable at the end of 10 years.

Calculate the probability that the benefit reserve at the end of 10 years will not be sufficient to cover L_{10}.

$${}_{10}V = 100\left(1 - \frac{\ddot{a}_{60}}{\ddot{a}_{50}}\right) = 27.27273$$

Here $K = K_{60}$, the curtate future lifetime of (60)

$$L_{10} > {}_{10}V \iff 100v^{K+1} - P\ddot{a}_{\overline{K+1|}} > {}_{10}V$$

Call this a!

$$\iff \cancel{\left(100 + \frac{P}{d}\right)} v^{K+1} > \boxed{\frac{{}_{10}V + \frac{P}{d}}{100 + \frac{P}{d}}}$$

$$\iff K+1 < \frac{\log(a)}{-\delta}$$

$$\iff K < \frac{\log(a)}{-\delta} - 1 = \frac{\log\left(\overset{.6923077}{.\cancel{6289007}}\right)}{-\log(1.04)} - 1$$

$$= 8.37578$$

$$\Pr[L_{10} > {}_{10}V] = \Pr[K < 8.37578] = \Pr[K \le 8]$$

$$= {}_{9}q_{\underset{60}{\cancel{50}}} = 1 - {}_{9}p_{\underset{60}{\cancel{50}}} = 1 - .714 = \underline{\underline{.286}}$$

Question No. 10:

For a fully discrete whole life insurance of \$10 issued to (55), you are given:

- $i = 0.05$
- $A_{55} = 0.5768$
- $\ddot{a}_{55} = 8.8872$
- $A_{63} = 0.6771$
- ${}^2A_{63} = 0.4883$
- L_8 denotes the prospective loss random variable at the end of 8 years.

Calculate $\mathrm{Var}[L_8]$.

$$P = 10 \frac{A_{55}}{\ddot{a}_{55}} = 10 \frac{(.5768)}{8.8872} \approx 0.6490233$$

$$\mathrm{Var}[L_8] = \left(10 + \frac{P}{d}\right)^2 \left({}^2A_{63} - A_{63}^2\right)$$

$$= \left(10 + \frac{0.6490233}{.05/1.05}\right)^2 \left(0.4883 - 0.6771^2\right)$$

$$= \underline{\underline{16.65878}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$	$1000\,{}_{20}E_x$	x
0	10,000,000	20.42	16.8010	49.00	25.92	728.54	541.95	299.89	0
5	9,749,503	0.98	17.0379	35.59	8.45	743.89	553.48	305.90	5
10	9,705,588	0.85	16.9119	42.72	9.37	744.04	553.34	305.24	10
15	9,663,731	0.91	16.7384	52.55	11.33	743.71	552.69	303.96	15
20	9,617,802	1.03	16.5133	65.28	14.30	743.16	551.64	301.93	20
21	9,607,896	1.06	16.4611	68.24	15.06	743.01	551.36	301.40	21
22	9,597,695	1.10	16.4061	71.35	15.87	742.86	551.06	300.82	22
23	9,587,169	1.13	16.3484	74.62	16.76	742.68	550.73	300.19	23
24	9,576,288	1.18	16.2878	78.05	17.71	742.49	550.36	299.49	24
25	9,565,017	1.22	16.2242	81.65	18.75	742.29	549.97	298.73	25
26	9,553,319	1.27	16.1574	85.43	19.87	742.06	549.53	297.90	26
27	9,541,153	1.33	16.0873	89.40	21.07	741.81	549.05	297.00	27
28	9,528,475	1.39	16.0139	93.56	22.38	741.54	548.53	296.01	28
29	9,515,235	1.46	15.9368	97.92	23.79	741.24	547.96	294.92	29
30	9,501,381	1.53	15.8561	102.48	25.31	740.91	547.33	293.74	30
31	9,486,854	1.61	15.7716	107.27	26.95	740.55	546.65	292.45	31
32	9,471,591	1.70	15.6831	112.28	28.72	740.16	545.90	291.04	32
33	9,455,522	1.79	15.5906	117.51	30.63	739.72	545.07	289.50	33
34	9,438,571	1.90	15.4938	122.99	32.68	739.25	544.17	287.82	34
35	9,420,657	2.01	15.3926	128.72	34.88	738.73	543.18	286.00	35
36	9,401,688	2.14	15.2870	134.70	37.26	738.16	542.11	284.00	36
37	9,381,566	2.28	15.1767	140.94	39.81	737.54	540.92	281.84	37
38	9,360,184	2.43	15.0616	147.46	42.55	736.86	539.63	279.48	38
39	9,337,427	2.60	14.9416	154.25	45.48	736.11	538.22	276.92	39
40	9,313,166	2.78	14.8166	161.32	48.63	735.29	536.67	274.14	40
41	9,287,264	2.98	14.6864	168.69	52.01	734.40	534.99	271.12	41
42	9,259,571	3.20	14.5510	176.36	55.62	733.42	533.14	267.85	42
43	9,229,925	3.44	14.4102	184.33	59.48	732.34	531.12	264.31	43
44	9,198,149	3.71	14.2639	192.61	63.61	731.17	528.92	260.48	44
45	9,164,051	4.00	14.1121	201.20	68.02	729.88	526.52	256.34	45
46	9,127,426	4.31	13.9546	210.12	72.72	728.47	523.89	251.88	46
47	9,088,049	4.66	13.7914	219.36	77.73	726.93	521.03	247.08	47
48	9,045,679	5.04	13.6224	228.92	83.06	725.24	517.91	241.93	48
49	9,000,057	5.46	13.4475	238.82	88.73	723.39	514.51	236.39	49
50	8,950,901	5.92	13.2668	249.05	94.76	721.37	510.81	230.47	50
51	8,897,913	6.42	13.0803	259.61	101.15	719.17	506.78	224.15	51
52	8,840,770	6.97	12.8879	270.50	107.92	716.76	502.40	217.42	52
53	8,779,128	7.58	12.6896	281.72	115.09	714.12	497.64	210.27	53
54	8,712,621	8.24	12.4856	293.27	122.67	711.24	492.47	202.70	54
55	8,640,861	8.96	12.2758	305.14	130.67	708.10	486.86	194.72	55
56	8,563,435	9.75	12.0604	317.33	139.11	704.67	480.79	186.32	56
57	8,479,908	10.62	11.8395	329.84	147.99	700.93	474.22	177.53	57
58	8,389,826	11.58	11.6133	342.65	157.33	696.85	467.12	168.37	58
59	8,292,713	12.62	11.3818	355.75	167.13	692.41	459.46	158.87	59
60	8,188,074	13.76	11.1454	369.13	177.41	687.56	451.20	149.06	60
61	8,075,403	15.01	10.9041	382.79	188.17	682.29	442.31	139.00	61
62	7,954,179	16.38	10.6584	396.70	199.41	676.56	432.77	128.75	62
63	7,823,879	17.88	10.4084	410.85	211.13	670.33	422.54	118.38	63
64	7,683,979	19.52	10.1544	425.22	223.34	663.56	411.61	107.97	64
65	7,533,964	21.32	9.8969	439.80	236.03	656.23	399.94	97.60	65

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$	$1000\,{}_{20}E_x$	x
66	7,373,338	23.29	9.6362	454.56	249.20	648.27	387.53	87.37	66
67	7,201,635	25.44	9.3726	469.47	262.83	639.66	374.36	77.38	67
68	7,018,432	27.79	9.1066	484.53	276.92	630.35	360.44	67.74	68
69	6,823,367	30.37	8.8387	499.70	291.46	620.30	345.77	58.54	69
70	6,616,155	33.18	8.5693	514.95	306.42	609.46	330.37	49.88	70
71	6,396,609	36.26	8.2988	530.26	321.78	597.79	314.27	41.86	71
72	6,164,663	39.62	8.0278	545.60	337.54	585.25	297.51	34.53	72
73	5,920,394	43.30	7.7568	560.93	353.64	571.81	280.17	27.96	73
74	5,664,051	47.31	7.4864	576.24	370.08	557.43	262.31	22.19	74
75	5,396,081	51.69	7.2170	591.49	386.81	542.07	244.03	17.22	75
76	5,117,152	56.47	6.9493	606.65	403.80	525.71	225.46	13.04	76
77	4,828,182	61.68	6.6836	621.68	421.02	508.35	206.71	9.61	77
78	4,530,360	67.37	6.4207	636.56	438.42	489.97	187.94	6.88	78
79	4,225,163	73.56	6.1610	651.26	455.95	470.57	169.31	4.77	79
80	3,914,365	80.30	5.9050	665.75	473.59	450.19	151.00	3.19	80
81	3,600,038	87.64	5.6533	680.00	491.27	428.86	133.19	2.05	81
82	3,284,542	95.61	5.4063	693.98	508.96	406.62	116.06	1.27	82
83	2,970,496	104.28	5.1645	707.67	526.60	383.57	99.81	0.75	83
84	2,660,734	113.69	4.9282	721.04	544.15	359.79	84.59	0.42	84
85	2,358,246	123.89	4.6980	734.07	561.57	335.40	70.56	0.22	85
86	2,066,090	134.94	4.4742	746.74	578.80	310.56	57.83	0.11	86
87	1,787,299	146.89	4.2571	759.03	595.79	285.44	46.50	0.05	87
88	1,524,758	159.81	4.0470	770.92	612.51	260.21	36.61	0.02	88
89	1,281,083	173.75	3.8442	782.41	628.92	235.11	28.17	0.01	89
90	1,058,491	188.77	3.6488	793.46	644.96	210.36	21.13	0.00	90
91	858,676	204.93	3.4611	804.09	660.61	186.21	15.41	0.00	91
92	682,707	222.27	3.2812	814.27	675.83	162.90	10.91	0.00	92
93	530,959	240.86	3.1091	824.01	690.59	140.69	7.47	0.00	93
94	403,072	260.73	2.9450	833.30	704.86	119.79	4.93	0.00	94
95	297,981	281.91	2.7888	842.14	718.61	100.43	3.13	0.00	95
96	213,977	304.45	2.6406	850.53	731.83	82.78	1.90	0.00	96
97	148,832	328.34	2.5002	858.48	744.50	66.97	1.10	0.00	97
98	99,965	353.60	2.3676	865.99	756.60	53.09	0.60	0.00	98
99	64,617	380.20	2.2426	873.06	768.13	41.14	0.31	0.00	99
100	40,049	408.12	2.1252	879.70	779.08	31.12	0.15	0.00	100
101	23,705	437.28	2.0152	885.93	789.44	22.91	0.07	0.00	101
102	13,339	467.61	1.9123	891.76	799.21	16.37	0.03	0.00	102
103	7,101	498.99	1.8164	897.19	808.41	11.33	0.01	0.00	103
104	3,558	531.28	1.7273	902.23	817.02	7.56	0.00	0.00	104
105	1,668	564.29	1.6447	906.90	825.06	4.86	0.00	0.00	105
106	727	597.83	1.5685	911.22	832.53	2.99	0.00	0.00	106
107	292	631.64	1.4984	915.19	839.46	1.76	0.00	0.00	107
108	108	665.45	1.4341	918.82	845.84	0.98	0.00	0.00	108
109	36	698.97	1.3755	922.14	851.69	0.52	0.00	0.00	109
110	11	731.87	1.3223	925.15	857.04	0.26	0.00	0.00	110