MATH 3630 - Actuarial Mathematics I
Fall 2012 - Valdez
Homework No. 6
due Wednesday, 9:30 PM, 5 December 2012

Please return this page with your signature. Please write your name and student number at the spaces provided:

Name: SUGGESTED SOLUTIONS **Student ID:** _____

I certify that this is my own work, and that I have not copied the work of another student.

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Circle your class lecture: 3–4:15 PM 5–6:15 PM

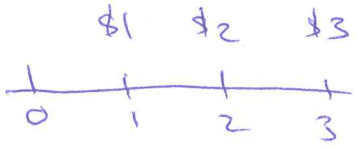
For a special 3-year temporary life annuity on (65), you are given:

- The annuity payments are \$1, \$2, and \$3, respectively, payable at the end of each year while (65) is alive. No further payments made after 3 years.
- Mortality is based on the following extract from a life table:

deaths 100 200 300

x	65	66	67	68
ℓ_x	9500	9400	9200	8900

- $i = 5\%$



Calculate the following:

(a) the actuarial present value of this annuity;

(b) the variance of the present value random variable of this annuity; and

(c) the probability that the total present value of payments will be (strictly) less than \$3.

Use first principles to compute mean & variance (note payments are at e.o.y.)

K	$\Pr[K_{65}=k]$	y (p.v. of benefits)	$y * \Pr[K_{65}=k]$	$y^2 * \Pr[K_{65}=k]$
0	100/9500	ϕ	0	0
1	200/9500	$v = 1/1.05 = 0.952381$	0.02005013	0.01909536
2	300/9500	$v+2v^2 = 2.766440$	0.08736126	0.24167968
≥ 3	8900/9500	$v+2v^2+3v^3 = 5.357953$	5.01955569	26.89454200
		Sum	5.126967	27.15532

(a) $APV(\text{annuity}) = E[Y] = \sum y * Pr[K_{65} = k] = \underline{5.126967}$

(b) $Var[Y] = \underbrace{\sum y^2 * Pr[K_{65} = k]}_{27.15532} - (5.126967)^2$

$= \underline{0.8695256}$

(c) $Pr[Y < 3] = Pr[K_{65} \leq 2]$

↙ Clearly PV is < 3 if death is on or before 2 years

$= {}_3q_{65} = 1 - {}_3p_{65}$

$= 1 - \frac{l_{68}}{l_{65}}$

$= 1 - \frac{8900}{9500}$

$= 600/9500 = 0.06315789$

↘ Just to confirm, if you use the CPT to evaluate APV, we get

$$vp_{65} + 2v^2 p_{65} p_{66} + 3v^3 p_{65} p_{66} p_{67}$$

$$= \frac{v l_{66} + 2v^2 l_{67} + 3v^3 l_{68}}{l_{65}}$$

$$= \frac{\frac{1}{1.05}(9400) + 2\frac{1}{1.05^2}(9200) + 3\frac{1}{1.05^3}(8900)}{9500}$$

$= \underline{5.126967}$ exactly the same result !!