MATH 3630 - Actuarial Mathematics I
Fall 2015 - Valdez
Homework No. 5
due Wednesday, 5:00 PM, 2 December 2015

Please return this page with your signature. Please write your name and student number at the spaces provided:

Name: Suggested Solutions **Student ID:** ____________

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An insurance company sells one-year term insurance policies to n policyholders, all with independent future lifetimes. Each policyholder is age x and pays \$20 now to receive the coverage.

- The death benefit of \$1,000 is payable at the end of the first year, if death occurs during the first year.
- $i = 5\%$
- $q_x = 0.01$
- The 95th percentile of the standard normal distribution is 1.645.
- The 99th percentile of the standard normal distribution is 2.326.

Based on the normal approximation, calculate the smallest n such that the total payments received now from all policyholders will be sufficient to pay present value of all claims:

(a) with probability of at least 0.95;

(b) with probability of at least 0.99; and

(c) Intuitively explain why one is larger than the other.

Let Y be the PV of all claims.

$$E[Y] = 1000\, v\, q_x \cdot n = 1000\left(\frac{1}{1.05}\right)(.01) \cdot n = 9.52381n$$

$$Var[Y] = n \cdot 1000^2 v^2 q_x(1-q_x) = 8979.592n$$

(a) $\Pr[Y \leq 20n] \geq .95 \Rightarrow \Pr\left[\text{Normal} \leq \dfrac{20n - 9.52381n}{\sqrt{8979.592n}}\right] \geq .95$

$$\Rightarrow 10.47619n \geq 1.645\sqrt{8979.592n} \Rightarrow n \geq \left(\frac{1.645\sqrt{8979.592}}{10.47619}\right)^2 = 221.4021$$

Choose $n = 222$ policies

(b) Similarly, $10.47619n \geq 2.326\sqrt{8979.592n}$

$$\Rightarrow \quad n \geq \left(\frac{2.326\sqrt{8979.592}}{10.47619} \right)^2 = 442{,}659$$

Choose $n = 443$

(c) Spreading the risk of having more claims to a larger pool of individuals helps reduce the variability of claims per individual. More individuals then increase the likelihood of having sufficient funds to cover claims. This concept is akin to the "law of large numbers" which is a fundamental concept of "insurance pooling of risks".