

MATH 3630 - Actuarial Mathematics I
Fall 2008 - Valdez
Homework No. 5
due Wednesday, 6:50 PM, October 29, 2008

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Affected by the financial crisis, a major corporation has to layoff 1,000 of its employees. The company offers severance package to each laid-off employee a sum of \$100 per year for life, payable annually, to start immediately at the beginning of each year.

The company creates a fund immediately to set aside to pay for the cost of the severance packages. Assume the following:

- all employees laid-off are each age 40;
- mortality follows the Illustrative Life Table with discount rate $i = 6\%$;
- the employees' lifetimes are independent; and
- the fund is determined, using Normal approximation, such that the probability the fund is sufficient to make all payments is 0.95.

Calculate the amount of the initial fund the company must set aside.

SOLUTIONS TO HW5

EMIL VALDEZ

$$\text{Let } Y = \text{PV of all benefit payments} \\ = 100Y_1 + \dots + 100Y_{1000}$$

where $Y_i = \text{PV of a whole life annuity-due to } (40)$
of \$1 each year, $i = 1, \dots, 1000$

$$\text{Clearly, } EY_i = \ddot{A}_{40} = 14.8166$$

$$\text{and } \text{Var} Y_i = \frac{{}^2A_{40} - A_{40}^2}{d^2} = \frac{\frac{48.63}{1000} - \left(\frac{161.32}{1000}\right)^2}{(0.06/1.06)^2} \\ = 7.0553933$$

$$\text{Thus, } EY = 100(14.8166) * 1000 = 1,481,660$$

and by independence,

$$\text{Var} Y = 100^2 (7.0553933) * 1000 = 70,553,933$$

If $F = \text{required initial fund}$, then

$$P(Y \leq F) = P\left(\frac{Y - EY}{\sqrt{\text{Var} Y}} \leq \frac{F - 1481660}{\sqrt{70553933}}\right) = 0.95$$

1.645, by Normal approximation

Therefore,

$$F = 1481660 + 1.645 \sqrt{70553933}$$

$$= ~~1,481,660~~ \quad \underline{\underline{1,495,477.41}}$$