MATH 3630 - Actuarial Mathematics I
Fall 2011 - Valdez
Homework No. 4
due Monday, 5:00 PM, 31 October 2011

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Joshua, currently age 40, just joined ABC company with a starting salary of $75,000 per year. ABC provides a benefit, payable at the moment of his death, equal to 4 times his salary if he dies while employed with the company and before reaching retirement age 65. Assume that Joshua intends to work for the company until retirement age 65.

You are given:

- Mortality follows de Moivre's law with $\omega = 100$.
- $\delta = 7\%$

1. Suppose that Joshua's salary increases continuously at an annual rate of 4%, that is, his salary at time $t$ from start of employment is

$$75000\, e^{0.04t}, \quad \text{for } t \geq 0.$$

   (a) Express the present value random variable of Joshua's death benefits.

   (b) Calculate the actuarial present value of his death benefits.

   (c) Calculate the variance of the present value of his death benefits.

2. Now, suppose that his salary rather increases compoundedly at an annual rate of 4% at the end of each year, that is, his salary at time $k$ from start of employment is

$$75000\,(1.04)^k, \quad \text{for } k = 0, 1, 2, \ldots$$

   His death benefit is still payable at the moment of death.

   (a) Calculate the actuarial present value of his death benefits.

   (b) Calculate the variance of the present value of his death benefits.

3. Explain the difference between the two actuarial present values.

First, note that Joshua's future lifetime, $T_{40} = T$, has density $f_{40}(t) = \frac{1}{60}$, $t \leq 60$, by de Moivre's law

① (a) Let Z be the present value r.v. of \$1 (or unit) continuously increasing at the rate of .04. Thus

$$Z = \begin{cases} e^{0.04T} e^{-.07T} = e^{-.03T}, & T \leq 25 \\ 0, & T > 25 \end{cases}$$

Thus, the present value r.v. of his benefit is

$$4 * 75{,}000\, Z = 300{,}000\, e^{-.03T}\, I(T \leq 25)$$

(b) $APV(\text{benefits}) = 300{,}000\, E[Z] = 300{,}000 \int_0^{25} e^{-.03t} \frac{1}{60}\, dt$

$$= \frac{300{,}000}{60} \left(\frac{1}{.03}\right)\left(1 - e^{-.03(25)}\right)$$

$$= \underline{\underline{87{,}938.9}}$$

(c) $Var[PV] = 300{,}000^2 \left\{ E[Z^2] - E[Z]^2 \right\}$

$$E[Z^2] = \int_0^{25} e^{-.06t} \frac{1}{60}\, dt = \frac{1}{60} \frac{1}{.06}\left(1 - e^{-.06(25)}\right)$$

$$= .2157972$$

$$Var[PV] = 300{,}000^2 (.2157972) - (87938.9)^2$$

$$= 11{,}688{,}495{,}863$$

② (a) The p.v. r.v. in this case can be expressed as

$$300{,}000\, (1.04)^{\lfloor T \rfloor}\, e^{-.07T}\, I(T \leq 25)$$

where $K = \lfloor T \rfloor$ curtate future lifetime of (40)

$$\text{APV (benefits)} = 300{,}000 \int_0^{25} (1.04)^{\lfloor t \rfloor} e^{-.07t} \frac{1}{60} dt$$

$$= \frac{300{,}000}{60} \sum_{K=0}^{24} \int_K^{K+1} (1.04)^K e^{-.07t} dt$$

$$= \frac{300{,}000}{60} \sum_{K=0}^{24} (1.04)^K \frac{1}{.07} \left( e^{-.07K} - e^{-.07(K+1)} \right)$$

$$= \frac{300{,}000}{60} \frac{1}{.07} \left(1 - e^{-.07}\right) \underbrace{\sum_{K=0}^{24} \left(1.04 e^{-.07}\right)^K}_{\frac{1-(1.04e^{-.07})^{25}}{1-(1.04e^{-.07})}}$$

$$= \underline{85{,}513.76}$$

(b) $$E[PV^2] = \frac{(300{,}000)^2}{60} \sum_{K=0}^{24} (1.04)^{2K} \int_K^{K+1} e^{-.14t} dt$$

$$= \frac{(300{,}000)^2}{60} \frac{1}{.14} \left(1 - e^{-.14}\right) \underbrace{\sum_{K=0}^{24} \left((1.04)^2 e^{-.14}\right)^K}_{\frac{1-((1.04)^2 e^{-.14})^{25}}{1-((1.04)^2 e^{-.14})}}$$

$$= 18{,}413{,}846{,}602$$

$$Var[PV] = 18413846602 - (85513.76)^2 = 11{,}101{,}243{,}453$$

3 When salary increases continuously, rather than in yearly intervals, the benefit amount at the moment of death will always be larger. This leads to a greater APV in 1 (b) than in 2 (a).