

MATH 3630 - Actuarial Mathematics I
Fall 2010 - Valdez
Homework No. 4
due Wednesday, 6:15 PM, 10 November 2010

Please return this page with your signature. Please write your name and student number at the spaces provided:

Name: EMIL Student ID: SUGGESTED SOLUTIONS

I certify that this is my own work, and that I have not copied the work of another student.

Signature: _____ Date: _____

You are currently exact age 55 and you just inherited an amount of \$1,000,000. You decided to immediately retire from work and invest your inheritance. Assume you will have no other source of income.

You have two possible investment strategies:

- Buy a policy with an annual benefit of \$60,000 payable, while alive, at the end of each year and a bequest amount ¹ of B_1 payable at the end of the year of your death. Assume you spend the full annual benefit each year you are alive and no amount is reinvested.
- Invest the full \$1,000,000 at the rate of 10% in the first year, spend \$60,000 at the end of that year and at that time, invest the difference by buying a policy with an annual benefit of \$60,000 payable, while alive, at the end of each year and a bequest amount of B_2 payable at the end of the year of your death. Again assume the full annual benefit each year is spent and no amount is reinvested. However, there is a 5% probability that you may lose 10% of your investment in the first year in which case no return on investment is received at the end of the first year and no amount to spend at the end of that year if alive.

Assume your mortality follows the *Illustrative Life Table* with interest rate of 6%.

1. Calculate the amount of B_1 so that the APV of the benefits of your policy is \$1,000,000.
2. Calculate the expected bequest amount B_2 in the second strategy.
3. Which strategy would you prefer? Justify your answer.

¹A bequest is an amount you leave to your estate for your loved ones.

Strategy 1

$$\begin{aligned} \text{APV}(\text{benefits}) &= 60,000 a_{55} + B_1 A_{55} \\ &= 60,000(12.2758 - 1) + B_1 \left(\frac{305.14}{1000} \right) \\ &= 1,000,000 \end{aligned}$$

$$\Rightarrow B_1 = \frac{1,000,000 - 60,000(11.2758)}{305.14/1000} = \underline{\underline{1,060,012}}$$

Strategy 2

Two possible scenarios: you lose 10% or not

Scenario A no loss of 10% in the first year

$$\begin{aligned} \text{APV}(\text{benefits}) &= 60,000 a_{56} + B_2 A_{56} \\ &= 1,040,000 \end{aligned}$$

$$\Rightarrow B_2 = \frac{1,040,000 - 60,000(11.0604)}{.31733} = 1,186,071$$

Scenario B loss of 10% in the first year

$$\begin{aligned} \text{APV}(\text{benefits}) &= 60,000 a_{56} + B_2 A_{56} \\ &= 900,000 \end{aligned}$$

$$\Rightarrow B_2 = \frac{900,000 - 60,000(11.0604)}{.31733} = 744,890.2$$

The expected bequest amount is

$$1,186,071(.95) + 744,890.2(.05) = \underline{\underline{1,164,012}}$$

For strategy 2, the APV of the benefits is equal to

$$\frac{(1,040,000 * .95 + 900,000 * .05) v P_{55}}{1.06 \left(1 - \frac{8.96}{1000}\right)} + \frac{60000 * .95 * v}{1.06}$$

$$= 1,019,570$$

This gives a higher APV than strategy 1 so that you are better off (actuarially) with strategy 2.