

Hw 3
p. 1

MATH 3630 - Actuarial Mathematics I
Fall 2015 - Valdez
Homework No. 3
due Wednesday, 5:00 PM, 21 October 2015

Please return this page with your signature. Please write your name and student number at the spaces provided:

Name: Suggested Solutions Student ID: _____

I certify that this is my own work, and that I have not copied the work of another student.

Signature: _____ Date: _____

For a whole life insurance of a benefit of 100 on (x) payable at the moment of death, you are given:

$$\mu_{x+t} = \begin{cases} 0.004, & \text{for } 0 < t \leq 10 \\ 0.005, & \text{for } 10 < t \leq 20 \\ 0.006, & \text{for } t > 20 \end{cases}$$

and

$$\delta_t = \begin{cases} 0.02, & \text{for } 0 < t \leq 20 \\ 0.05, & \text{for } t > 20 \end{cases}$$

- (1 point) Express the Present Value random variable for this life insurance (note the benefit is 100). You may write this as the random variable Z .
- (4 points) Calculate the Actuarial Present Value (APV) of the benefit for this insurance, i.e. $E[Z]$.
- (5 points) Calculate the variance of Z .

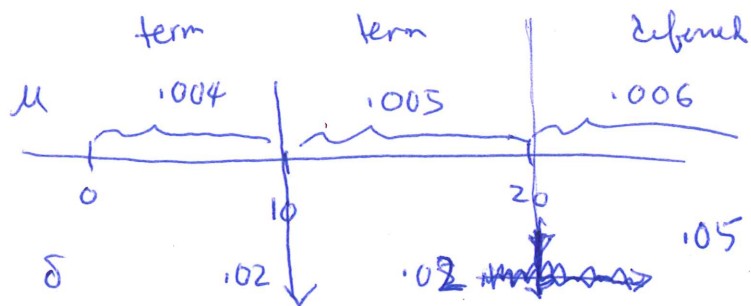
(1) $Z = 100 v^T = 100 e^{-\int_0^T \delta_s ds}$

$$= 100 \begin{cases} e^{-0.02T}, & T \leq 20 \\ e^{0.6 - 0.05T}, & T > 20 \end{cases}$$

\uparrow

$$e^{-\int_0^{20} 0.02 ds} \quad e^{-\int_{20}^T 0.05 ds}$$

HW No. 3
P. 2



(2) APV (insurance)

$$= 100 * \left[\frac{.004}{.024} (1 - e^{-.024(10)}) + e^{-.024(10)} \cdot \frac{.005}{.025} (1 - e^{-.025(10)}) \right. \\ \left. + e^{-.024(10)} e^{-.025(10)} \cdot \frac{.006}{.056} \right]$$

= ~~13,600.09~~ 13,600.09

$$(3) E[Z^2] = APV @ 2\delta = 100^2 * \left[\frac{.004}{.044} (1 - e^{-.044(10)}) + \right. \\ \left. e^{-.044(10)} \cdot \frac{.005}{.045} (1 - e^{-.045(10)}) + \right. \\ \left. e^{-.044(10)} e^{-.045(10)} \cdot \frac{.006}{.106} \right]$$

= ~~29,274.8~~ 815.3618

$$\text{Var}(Z) = E[Z^2] - (E[Z])^2 \\ = 815.3618 - (\frac{13,600.09}{100})^2 \\ = ~~629,274.8~~ 630.3994$$

corrected 10/22/2015
EW
Thanks to RW!