MATH 3630 - Actuarial Mathematics I
Fall 2012 - Valdez
Homework No. 3
due Wednesday, 6:15 PM, 17 October 2012

Please return this page with your signature. Please write your name and student number at the spaces provided:

Name: SUGGESTED SOLUTIONS **Student ID:** _____

I certify that this is my own work, and that I have not copied the work of another student.

Signature: _____ **Date:** _____

Circle your class lecture: 3–4:15 PM 5–6:15 PM

You are given the following extract from a select and ultimate life table:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
49	–	–	92,250	51
50	–	–	91,700	52
51	–	–	91,050	53
52	–	–	90,300	54

The following relationships also hold for all x:

- $q_{[x]} = 0.70 \times q_{[x-1]+1}$
- $q_{[x]+1} = 0.80 \times q_{x+1}$

$$\Rightarrow \quad 1 - \frac{\ell_{x+2}}{\ell_{[x]+1}} = 0.80\left(1 - \frac{\ell_{x+2}}{\ell_{x+1}}\right)$$

Solving for $\ell_{[x]+1}$, we get

$$\ell_{[x]+1} = \frac{1}{\frac{0.20}{\ell_{x+2}} + \frac{0.80}{\ell_{x+1}}}$$

Calculate the following:

1. ${}_3p_{[51]}$
2. $\ell_{[52]+1}$

$$\ell_{[52]+1} = \frac{1}{\frac{0.20}{90300} + \frac{0.80}{91050}} = 90,899$$

$$\ell_{[50]+1} = \frac{1}{\frac{0.20}{\ell_{52}} + \frac{0.80}{\ell_{51}}} = \frac{1}{\frac{0.20}{91700} + \frac{0.80}{92250}} = 92,139.47$$

$$\ell_{[51]+1} = \frac{1}{\frac{0.20}{91050} + \frac{0.80}{91700}} = 91,569.26$$

From $q_{[x]} = 0.70 * q_{[x-1]+1} \Rightarrow 1 - \frac{l_{[x]+1}}{l_{[x]}} = 0.70\left(1 - \frac{l_{x+1}}{l_{[x-1]+1}}\right)$

Solving for $l_{[x]}$, we get

$$l_{[x]} = \frac{l_{[x]+1}}{0.30 + 0.70\,\frac{l_{x+1}}{l_{[x-1]+1}}}$$

Thus, for $x = 51$, we have

$$l_{[51]} = \frac{l_{[51]+1}}{0.30 + 0.70\,\frac{l_{52}}{l_{[50]+1}}} = \frac{91,569.26}{0.30 + 0.70\left(\frac{91,700}{92,139.47}\right)}$$

$$= \frac{91569.26}{0.9966613} = 91,876.01$$

Finally we have

$$3p_{[51]} = \frac{l_{54}}{l_{[51]}} = \frac{90,300}{91,876.01} = \underline{\underline{0.9828464}}$$