MATH 3630 - Actuarial Mathematics I
Fall 2010 - Valdez
Homework No. 3
due Wednesday, 6:15 PM, 20 October 2010

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For a special whole life insurance on (x) payable at the moment of death, you are given:

- Mortality of (x) follows de Moivre's law.
- The probability that (x) survives another 24 years is 0.70.
- The force of interest is expressed as

$$\delta_s = \begin{cases} 0.05, & \text{for } 0 < s \leq 20 \\ 0.10, & \text{for } s > 20 \end{cases}$$

- The death benefit at time t is $b_t = 100(1.05)^t$, for $t > 0$.
- The present value random variable for this insurance at issue is denoted by Z.

1. (5 points) Calculate the Actuarial Present Value (APV) of the benefit for this insurance.

2. (5 points) Calculate the variance of Z.

If $X \sim \text{Uniform}(0,\omega)$, then $T_x \sim$ Uniform on $(0, \omega - x)$

Given $P(T_x > 24) = .70 = 1 - \frac{24}{\omega - x} \Rightarrow \omega - x = 24/.3 = 80$

Thus $T_x \sim$ Uniform on $(0, 80)$

The best approach to calculate mean and variance of Z is the explicitly write Z.

Note that $1.05 = e^{\log(1.05)}$

(1) $$Z = b_T v^T = \begin{cases} 100 * \left(e^{\log(1.05)-.05}\right)^T, & T \le 20 \\ 100 * \underbrace{e^{-20(.05)} e^{20(.10)}}_{e} \left(e^{\log(1.05)-.10}\right)^T, & 20 < T \le 80 \end{cases}$$

let $a = .05 - \log(1.05)$

$b = .10 - \log(1.05)$

$$APV = E(Z) = 100 * \left[\int_0^{20} e^{-at} \frac{1}{80} dt + e \int_{20}^{80} e^{-bt} \frac{1}{80} dt\right]$$

$$= \frac{100}{80} * \left[\underbrace{\frac{1}{a}\left(1 - e^{-20a}\right)}_{19.75997} + \underbrace{\frac{e}{b}\left(e^{-20b} - e^{-80b}\right)}_{18.17814}\right]$$

$$= \underline{47.42264}$$

(2) $$E(Z^2) = \frac{100^2}{80} * \left[\int_0^{20} e^{-2at} dt + e^2 \int_{20}^{80} e^{-2bt} dt\right]$$

$$= \frac{100^2}{80} * \left[\underbrace{\frac{1}{2a}\left(1 - e^{-40a}\right)}_{19.52378} + \underbrace{\frac{e^2}{2b}\left(e^{-40b} - e^{-160b}\right)}_{9.282556}\right]$$

$$= 3600.72$$

$$Var(Z) = 3600.72 - (47.42264)^2$$

$$= \underline{1,351.81}$$