MATH 3630 - Actuarial Mathematics I
Fall 2009 - Valdez
Homework No. 3
due Monday, 6:50 PM, 19 October 2009

Please return this page with your signature. Please write your name and student number at the spaces provided:

Name: SUGGESTED SOLUTIONS **Student ID:** EMIL

I certify that this is my own work, and that I have not copied the work of another student.

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For a whole life insurance of a benefit of 10 on (x) payable at the moment of death, you are given:

$$\mu_{x+t} = \begin{cases} 0.001, & \text{for } 0 < t \leq 20 \\ 0.002, & \text{for } t > 20 \end{cases}$$

and

$$\delta_t = \begin{cases} 0.04, & \text{for } 0 < t \leq 10 \\ 0.05, & \text{for } t > 10 \end{cases}.$$

1. (1 point) Express the Present Value random variable for this life insurance (note the benefit is equal to 10). You may write this as the random variable Z.

2. (4 points) Calculate the Actuarial Present Value (APV) of the benefit for this insurance.

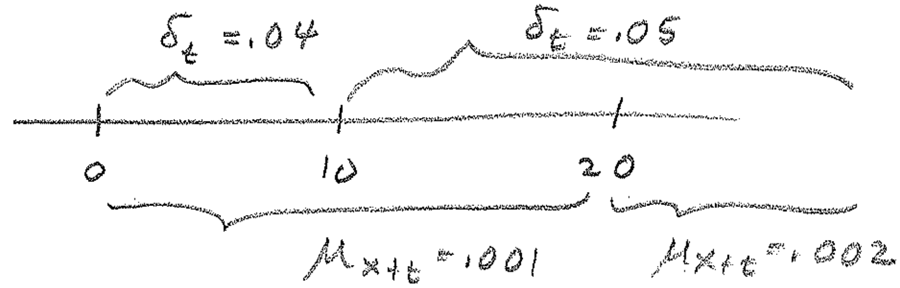
3. (5 points) Calculate the variance of Z.

(1) $Z = 10\, v_T = 10 \exp\left(-\int_0^T \delta_s\, ds\right)$

$$= 10 * \begin{cases} e^{-.04T}, & 0 < T \leq 10 \\ e^{.10 - .05T}, & T > 10 \end{cases}$$

$$e^{-\int_0^{10} .04 ds - \int_{10}^{T} .05 ds}$$

$$= e^{-.4 - .05(T-10)}$$



(2) APV (benefit) $= E(Z) = 10 \int_0^\infty v^t \, {}_tp_x \, \mu_{x+t} \, dt$

break $\int_0^\infty$ into 3 integrals

$$= 10 \Big[\int_0^{10} .001 \, e^{-.001t} e^{-.04t} \, dt$$

$$+ \int_{10}^{20} .001 \, e^{-.001t} e^{-.4} e^{-\int_{10}^{t} .05 \, ds} \, dt$$

$$+ \int_{20}^{\infty} .002 \, e^{-.02} e^{-\int_{20}^{t} .002 \, ds} e^{-.4} e^{-\int_{10}^{t} .05 \, ds} \, dt \Big]$$

$$= 10 \Big[\frac{.001}{.041}\left(1 - e^{-.41}\right) + \frac{.001 e^{.10}}{.051}\left(e^{-.51} - e^{-1.02}\right)$$

$$+ \frac{.002 \, e^{+.12}}{.052} e^{-1.04} \Big]$$

$$= \underline{0.2873}$$

(3) To get Var(Z), use $E(Z^2) - (E(Z))^2$ and same procedure above except δ's are replaced by 2δ's.

HW3
P2

$$E(Z^2) = 10^2 * \left[\frac{.001}{.081}\left(1 - e^{-.81}\right) + \frac{.001}{.101} e^{.2}\left(e^{-1.01} - e^{-2.02}\right) + \frac{.002}{.102} e^{.22} e^{-2.04} \right]$$

$$= \underline{1.2831}$$

Thus,

$$Var(Z) = 1.2831 - (.2873)^2$$

$$= \underline{\underline{1.2006}}$$

HW3
p.2