MATH 3630 - Actuarial Mathematics I
Fall 2008 - Valdez
Homework No. 3
due Wednesday, 6:50 PM, October 8, 2008

Please return this page with your signature. Please write your name and student number at the spaces provided:

Name: SUGGESTED SOLUTION **Student ID:** EMIL

I certify that this is my own work, and that I have not copied the work of another student.

Signature: _____ **Date:** _____

For a whole life insurance of a benefit of 1,000 on (x) payable at the moment of death, you are given:

$$\mu_{x+t} = \begin{cases} 0.004, & \text{for } 0 < t \leq 10 \\ 0.005, & \text{for } t > 10 \end{cases}$$

and

$$\delta_t = \begin{cases} 0.03, & \text{for } 0 < t \leq 20 \\ 0.02, & \text{for } t > 20 \end{cases}$$

1. (1 point) Express the Present Value random variable for this life insurance (note the benefit is 1,000). You may write this as the random variable Z.
2. (4 points) Calculate the Actuarial Present Value (APV) of the benefit for this insurance.
3. (5 points) Calculate the variance of Z.

(1) $Z = 1000 v^T = 1000 \exp\left(-\int_0^T \delta_s \, ds\right)$

$$= 1000 * \begin{cases} e^{-.03T}, & 0 < T \leq 20 \\ e^{-.2 - .02T}, & T > 20 \end{cases}$$

$$e^{-\int_0^{20} .03 \, ds - \int_{20}^T .02 \, ds}$$

(2) $APV(\text{benefit}) = E(Z) = \int_0^\infty 1000 v^t {}_tp_x \mu_{x+t}\, dt$

$$= 1000\Bigg[\int_0^{10} e^{-\int_0^t .03\,ds} e^{-\int_0^t .004\,ds} .004\,dt +$$

$$+ \int_{10}^{20} e^{-\int_0^t .03\,ds} e^{-\int_0^{10} .004\,ds - \int_{10}^t .005\,ds} .005\,dt$$

$$+ \int_{20}^{\infty} e^{-\int_0^{20} .03\,ds - \int_{20}^t .02\,ds} e^{-\int_0^{10} .004\,ds - \int_{10}^t .005\,ds} * .005\,dt \Bigg]$$

$$= 1000\Bigg[.004\left(\frac{1-e^{-.034(10)}}{.034}\right) + .005\, e^{-.04+.05}\left(\frac{e^{-.35}-e^{-.7}}{.035}\right)$$

$$+ .005\, e^{-.6-.04+.4+.05}\frac{e^{-.025(20)}}{.025}\Bigg]$$

$$= 1000\Big[\tfrac{2}{17}(1-e^{-.34}) + \tfrac{1}{7}e^{.01}(e^{-.35}-e^{-.7})$$

$$+ \tfrac{1}{5}e^{-.19}e^{-.5}\Big]$$

$$= 164.2523$$

(3) To get the variance of Z, same procedure above except the δ's are replaced by 2δ's when computing $E(Z^2)$

(3) $E(Z^2) = 1000^2 \left[.004 \frac{(1 - e^{-.064(10)})}{.064} \right.$

Constant is squared which variance is computed

$+ .005 e^{-.04+.05} \frac{(e^{-.65} - e^{-1.3})}{.065}$

$\left. + .005 e^{-1.2-.04+.8+.05} \frac{e^{-.045(20)}}{.045} \right]$

$= 1000^2 \left[\frac{1}{16}(1 - e^{-.64}) + \frac{1}{13} e^{+.01}(e^{-.65} - e^{-1.3}) \right.$

$\left. + \frac{1}{9} e^{-.39} e^{-.9} \right]$

$= \underline{79,516.15}$

$Var(Z) = E(Z^2) - (E(Z))^2$

$= 79516.15 - (164.2523)^2$

$= \underline{52,537.33}$