

MATH 3630 - Actuarial Mathematics I  
Fall 2012 - Valdez  
Homework No. 2  
due Wednesday, 6:15 PM, 26 September 2012

Please return this page with your signature. Please write your name and student number at the spaces provided:

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Circle your class lecture: 3-4:15 PM    5-6:15 PM

In a group of population consisting of half female and half male at birth, you are given that the survival function for female is

$$S_0^f(x) = \left(1 - \frac{x}{100}\right)^{1/3}, \quad \text{for } 0 \leq x \leq 100,$$

while that for male is

$$S_0^m(x) = \left(1 - \frac{x}{90}\right)^{1/2}, \quad \text{for } 0 \leq x \leq 90.$$

The superscripts  $f$  and  $m$  are to refer to female and male, respectively.

1. Calculate the proportions of surviving male and female at age 45.
2. Calculate the probability that a randomly selected person from this group who has reached age 45 will survive another 20 years, but then dies the following 10 years.

① The probability of a newborn female surviving to 45 is

$$S_0^f(45) = \left(\frac{55}{100}\right)^{1/3} = \left(\frac{11}{20}\right)^{1/3}$$

and that of a newborn male is

$$S_0^m(45) = \left(\frac{45}{90}\right)^{1/2} = \left(\frac{1}{2}\right)^{1/2}$$

Thus, the proportion of surviving female is

$$\frac{0.5 \left(\frac{11}{20}\right)^{1/3}}{0.5 \left(\frac{11}{20}\right)^{1/3} + 0.5 \left(\frac{1}{2}\right)^{1/2}} = 0.5367572$$

and the proportion of surviving male is

$$\frac{0.5(1/2)^{1/2}}{0.5(1/20)^{1/3} + 0.5(1/2)^{1/2}} = 0.4632428$$

$$\begin{aligned} \textcircled{2} \text{ For a female, } {}_{20|10}q_{45}^f &= \frac{S_0^f(65) - S_0^f(75)}{S_0^f(45)} \\ &= \frac{(35/100)^{1/3} - (25/100)^{1/3}}{(55/100)^{1/3}} = \frac{35^{1/3} - 25^{1/3}}{55^{1/3}} \\ &= .09125767 \end{aligned}$$

$$\begin{aligned} \text{For a male, } {}_{20|10}q_{45}^m &= \frac{S_0^m(65) - S_0^m(75)}{S_0^m(45)} \\ &= \frac{(25/90)^{1/2} - (15/90)^{1/2}}{(45/90)^{1/2}} = \frac{25^{1/2} - 15^{1/2}}{45^{1/2}} \\ &= .1680057 \end{aligned}$$

For a randomly selected person,

$$\begin{aligned} {}_{20|10}q_{45} &= (.09125767)(0.5367572) + (.1680057)(0.4632428) \\ &= \underline{\underline{0.1268107}} \end{aligned}$$