MATH 3630 - Actuarial Mathematics I
Fall 2011 - Valdez
Homework No. 2
due Wednesday, 5:00 PM, 5 October 2011

Please return this page with your signature. Please write your name and student number at the spaces provided:

Name: SUGGESTED SOLUTIONS **Student ID:** ____________________

I certify that this is my own work, and that I have not copied the work of another student.

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Mortality for a population consisting of females and males follow a select-and-ultimate table, an extract of which is given below. Females have a 3-year select period while males have a 2-year select period. Assume mortality follows the Uniform Distribution of Death (UDD) between integral ages.

Females						Males				
x	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	$x+3$	x	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
50	80960	79827	78522	77025	53	50	70764	69124	67224	52
51	79530	78334	76958	75382	54	51	68823	67118	65146	53
52	78021	76760	75312	73655	55	52	66805	65036	62993	54
53	76430	75103	73581	71842	56	53	64711	62879	60768	55
54	74756	73362	71765	69944	57	54	62544	60651	58475	56
55	72998	71535	69863	67958	58	55	60305	58354	56117	57

1. Calculate the probability that a randomly chosen male from this population, now age 51.5 with select age 50, will die within the next 2 years and 3 months.

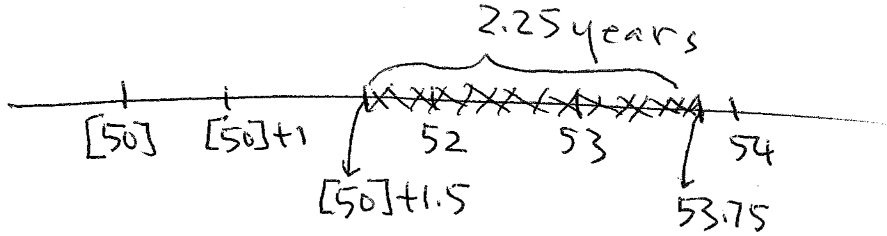
2. Calculate the probability that a randomly chosen female from this population, at select age 51, will die between the ages of 52.35 and 56.75.

3. Suppose that the composition of the population at select age 50 is 60% female and 40% male.

   (a) What is this composition after 5 years? (i.e. give the proportion of males and the proportion of females)

   (b) Calculate the probability that a randomly chosen person now age 55, with select age 50, from this population will survive the following year.

(1)

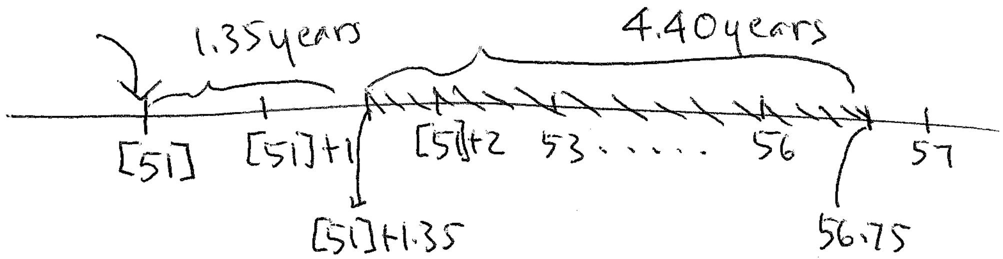


$${}_{2.25}q^{male}_{[50]+1.5} = 1 - {}_{2.25}p^{male}_{[50]+1.5} = 1 - \frac{l_{53.75}}{l_{[50]+1.5}}$$

$$= 1 - \frac{.25\,l_{53} + .75\,l_{54}}{.5(l_{[50]+1} + l_{52})} = 1 - \frac{.25(65146) + .75(62993)}{.5(69124 + 67224)}$$

$$= \underline{0.06810148}$$

(2)



$${}_{1.35|4.40}q^{female}_{[51]} = {}_{1.35}p^{female}_{[51]} - {}_{5.75}p^{female}_{[51]}$$

$$= \frac{l_{[51]+1.35} - l_{56.75}}{l_{[51]}}$$

$$= \frac{(.65\,l_{[51]+1} + .35\,l_{[51]+2}) - (.25\,l_{56} + .75\,l_{57})}{l_{[51]}}$$

$$= \frac{[.65(78334) + .35(76958)] - [.25(71842) + .75(69944)]}{79530}$$

$$= \underline{0.0934729}$$

(3) (a) $${}_5p_{[50]}^{female} = \frac{l_{55}}{l_{[50]}} = \frac{73655}{80960} = .9097703$$

$${}_5p_{[50]}^{male} = \frac{l_{55}}{l_{[50]}} = \frac{60768}{70764} = .8587417$$

at [50], 60% female & 40% male. After 5 years, we have

61.4%

$$\text{proportion female} = \frac{.60(.9097703)}{.60(.9097703)+.4(.8587417)} = .6137704$$

38.6%

$$\text{proportion male} = \frac{.40(.8587417)}{.60(.9097703)+.4(.8587417)} = .3862296$$

(b) $$p_{[50]+5}^{female} = \frac{l_{56}}{l_{55}} = 71842/73655$$

$$p_{[50]+5}^{male} = \frac{l_{56}}{l_{55}} = 58475/60768$$

For a randomly chosen person,

$$p_{[50]+5} = p_{[50]+5}^{female} * .6137704 + p_{[50]+5}^{male} * .3862296$$

$$= (71842/73655) * .6137704 + (58475/60768) * .3862296$$

$$= \underline{\underline{0.9703183}}$$