

MATH 3630 - Actuarial Mathematics I
Fall 2015 - Valdez
Homework No. 1
due Wednesday, 5:00 PM, 16 September 2015

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Let X be the lifetime (of a newborn) random variable with force of mortality defined by

$$\mu_x = \frac{1}{16\sqrt{x}}, \text{ for } x \geq 0.$$

1. Find an expression for the survival function $S_0(x)$.
2. Show that $S_0(x)$ is a valid survival function.
3. Calculate ${}_{20|10}q_{20}$ and interpret this probability.

$$\begin{aligned} (1) S_0(x) &= \exp\left[-\int_0^x \mu_z dz\right] = \exp\left[-\int_0^x \frac{1}{16} z^{-1/2} dz\right] \\ &= \exp\left[-\frac{1}{8} z^{1/2} \Big|_0^x\right] = e^{-\frac{1}{8}x^{1/2}} \end{aligned}$$

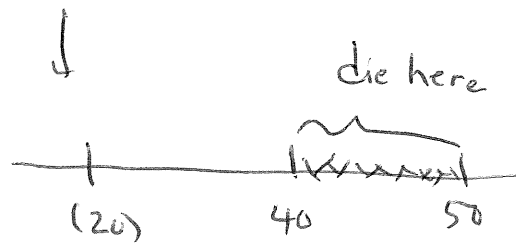
(2) For this to be a valid survival function, we must check:

$$i) S_0(0) = e^{-\frac{1}{8}(0)^{1/2}} = e^0 = 1$$

$$ii) \lim_{x \rightarrow \infty} S_0(x) = \lim_{x \rightarrow \infty} e^{-\frac{1}{8}x^{1/2}} = 0$$

$$iii) \text{non-increasing} \quad \frac{d}{dx} e^{-\frac{1}{8}x^{1/2}} = e^{-\frac{1}{8}x^{1/2}} \left(-\frac{1}{16}x^{-1/2}\right) < 0$$

(3) ${}_{20|10}q_{20}$ is the probability that a life (20) will survive for 20 years but die within the next 10 years after that



$$= \frac{S_0(40) - S_0(50)}{S_0(20)}$$

$$\approx \frac{e^{-\frac{1}{8}\sqrt{40}} - e^{-\frac{1}{8}\sqrt{50}}}{e^{-\frac{1}{8}\sqrt{20}}} \approx \underline{\underline{0.07067728}}$$