Michigan State University

STT 455 - Actuarial Models I

Fall 2013 semester

Homework No. 1

due Friday, 5:00 pm, September 20, 2013

Please follow the instructions below:

Return this page with your signature.

Submit your work to our graduate assistant, Ed Cruz, at C505 Wells.

Write your name and section number at the spaces provided:

Name: Suggested Solution **Section:** ______

I certify that this is my own work, and that I have not copied the work of another student.

Signature: ______ **Date:** ______

1. (30 points) Let T_0 be the lifetime of a newborn random variable with density function defined by

$$f_0(x) = \frac{1}{100}e^{-x/50} + \frac{1}{200}e^{-x/100}, \quad \text{for } x \geq 0.$$

Find expressions for the following:

(a) [7 points] $S_0(x)$

(b) [7 points] μ_x

(c) [7 points] ${}_tp_x$

(d) [9 points] Calculate ${}_{25|5}q_{40}$ and interpret this probability.

(a) $$S_0(x) = \int_x^\infty f_0(z)dz = \frac{1}{2}\int_x^\infty \frac{1}{50}e^{-z/50}dz + \frac{1}{2}\int_x^\infty \frac{1}{100}e^{-z/100}dz$$

$$= \frac{1}{2}e^{-x/50} + \frac{1}{2}e^{-x/100}$$

(b) $$\mu_x = \frac{f_0(x)}{S_0(x)} = \frac{\frac{1}{50}e^{-x/50} + \frac{1}{100}e^{-x/100}}{e^{-x/50} + e^{-x/100}}$$

(c) $${}_tp_x = \frac{S_0(x+t)}{S_0(x)} = \frac{e^{-(x+t)/50} + e^{-(x+t)/100}}{e^{-x/50} + e^{-x/100}}$$

40 65 70

(d) $${}_{25|5}q_{40} = {}_{25}p_{40} - {}_{30}p_{40}$$

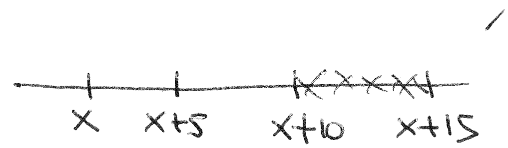
$$= \frac{e^{-65/50} + e^{-65/100}}{e^{-40/50} + e^{-40/100}} - \frac{e^{-70/50} + e^{-70/100}}{e^{-40/50} + e^{-40/100}}$$

$$= \underline{0.0459}$$

This gives the probability that a 40 year old will die between the ages of 65 and 70.

2. (30 points) You are given:

$$_{10}p_x = 0.90$$
$$_{15}p_x = 0.85$$
$$_{5}p_{x+5} = 0.95$$



Calculate the following:

(a) [10 points] $_{5}q_x$

(b) [10 points] $_{10|5}\,q_x$

(c) [10 points] $_{10}p_{x+5}$

(a) $_{10}p_x = {_5p_x} * {_5p_{x+5}} \Rightarrow {_5p_x} = \dfrac{_{10}p_x}{_5p_{x+5}} = \dfrac{0.90}{0.95}$

$\Rightarrow {_5q_x} = 1 - {_5p_x} = 1 - \dfrac{0.90}{0.95} = \dfrac{.05}{.95} = \dfrac{5}{95}$

$= \underline{\underline{.0526}}$

(b) $_{15}p_x = {_{10}p_x} * {_5p_{x+10}} \Rightarrow {_5p_{x+10}} = \dfrac{_{15}p_x}{_{10}p_x} = \dfrac{0.85}{0.90} = \dfrac{17}{18}$

$_{10|5}q_x = {_{10}p_x} * {_5q_{x+10}} = {_{10}p_x} * (1 - {_5p_{x+10}})$

$= 0.90 * \left(1 - \dfrac{17}{18}\right) = \dfrac{0.90}{18} = \underline{\underline{.05}}$

(c) $_{15}p_x = {_5p_x} * {_{10}p_{x+5}} \Rightarrow {_{10}p_{x+5}} = \dfrac{_{15}p_x}{_5p_x} = \dfrac{0.85}{0.90/0.95}$

$= \underline{\underline{0.8972}}$

3. (40 points) You are given the force of mortality:

$$\mu_x = \begin{cases} 0.02, & 0 < x < 50 \\ 0.04, & x \geq 50 \end{cases}$$

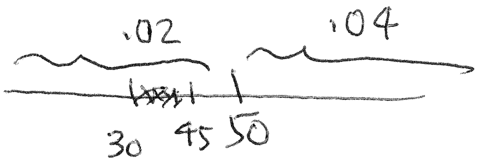
Calculate the following:

(a) [10 points] the probability that (30) will live another 15 years;

(b) [10 points] the probability that (65) will die before reaching age 75;

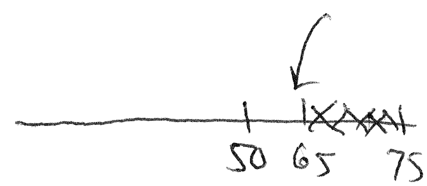
(c) [10 points] the probability that (40) will die between ages 45 and 55; and

(d) [10 points] the average lifetime of (40)



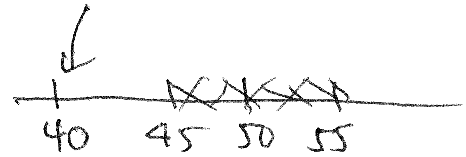
(a) $_{15}p_{30} = e^{-.02(15)} = e^{-.3}$

$= \underline{\underline{0.7408}}$



(b) $_{10}q_{65} = 1 - {}_{10}p_{65}$

$= 1 - e^{-.04(10)} = 1 - e^{-.4} = \underline{\underline{0.32968}}$

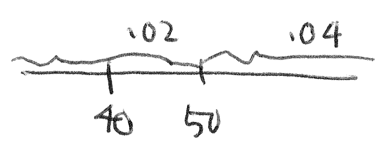


(c) $_{5|10}q_{40} = {}_5p_{40} * {}_{10}q_{45}$

$= {}_5p_{40} * (1 - {}_5p_{45} * {}_5p_{50}) = e^{-.02(5)} * (1 - e^{-.02(5)} e^{-.04(5)})$

$= e^{-.1} * (1 - e^{-.3}) = \underline{\underline{0.2345}}$

(d) $\mathring{e}_{40} = E[T_{40}] = \int_0^\infty {}_tp_{40}\,dt = \int_0^{10} {}_tp_{40}\,dt + \int_{10}^\infty {}_tp_{40}\,dt$



$= \int_0^{10} e^{-.02t}\,dt + e^{-.02(10)} \int_0^\infty e^{-.04t}\,dt = \frac{1}{.02}(1 - e^{-.2}) + e^{-.2}/.04 = 29.53173$