

MATH 3630 - Actuarial Mathematics I
 Fall 2012 - Valdez
 Homework No. 1
 due Monday, 6:15 PM, 17 September 2012

Please return this page with your signature. Please write your name and student number at the spaces provided:

Name: SUGGESTED SOLUTIONS Student ID: _____

I certify that this is my own work, and that I have not copied the work of another student.

Signature: _____ Date: _____

Circle your class lecture: 3-4:15 PM 5-6:15 PM

You are given the following survival function for a newborn:

$$S_0(x) = (1+x)e^{-x}, \quad \text{for } x \geq 0.$$

1. Demonstrate that $S_0(x)$ satisfies the important properties of a legitimate survival function.
2. Calculate μ_{50} .
3. Calculate ${}_{40}p_0$.
4. Give an expression for $S_{40}(t)$, the survival function for the future lifetime of (40).
5. Calculate the probability that (40) will reach centennial age 100.

$$(1) \quad S_0(0) = 1 \quad S_0(\infty) = \lim_{x \rightarrow \infty} (1+x)e^{-x} = \lim_{x \rightarrow \infty} \frac{1+x}{e^x} = \lim_{x \rightarrow \infty} \frac{1}{e^x} = 0$$

$$\frac{d}{dx} S_0(x) = e^{-x} + (1+x)(-e^{-x}) = -xe^{-x} \leq 0 \quad \text{for all } x \geq 0$$

\Rightarrow non-increasing

$$(2) \quad \mu_x = \frac{-1}{S_0(x)} \frac{d}{dx} S_0(x) = \frac{xe^{-x}}{(1+x)e^{-x}} = \frac{x}{1+x}$$

$$\mu_{50} = \frac{50}{51} = 0.9803922$$

$$(3) \quad {}_{40}p_0 = S_0(40) = (1+40)e^{-40} = 41e^{-40} \approx 0$$

Very slim chance
 of surviving to 40!

$$(4) S_{40}(t) = \frac{S_0(40+t)}{S_0(40)} = \frac{(41+t)e^{-40-t}}{41e^{-40}} = \left(\frac{41+t}{41}\right)e^{-t}, \quad t \geq 0$$

(5) Probability (40) will reach 100 is

$${}_{60}p_{40} = S_{40}(60) = \frac{41+60}{41} e^{-60} = \frac{101}{41} e^{-60} \approx 0$$

Even if you reach age 40, you also have a very slim chance of making it to 100!

Remarks:

Not surprising to find the slim probabilities in (3) & (5)

because:

(i) average age at death is only

$$2 = E[X]$$

$$= \int_0^{\infty} (1+x)e^{-x} dx$$

try this!

(ii)

| | |
|----------|----------------------------|
| <u>x</u> | <u>$S_0(x)$</u> |
|----------|----------------------------|

1 0.74

2 0.41

5 0.04

⋮

10 0.00

⋮

already after 5 years, you have
a very small chance of
surviving!