MATH 3630 - Actuarial Mathematics I
Fall 2011 - Valdez
Homework No. 1
due Wednesday, 5:00 PM, 21 September 2011

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For a certain population, the force of mortality is expressed as [1]

$$\mu_x = \log(2) + \frac{1}{2(80-x)}, \quad \text{for } 0 \le x < 80.$$

1. Derive the corresponding survival function $S_0(x)$ and demonstrate that it satisfies the important properties of a legitimate survival function.
2. Give an expression for ${}_tp_x$ and interpret this expression.
3. Calculate the probability that a life aged 45 will die between ages 60 and 70.

First note that $\int_0^x \mu_z dz = (\log 2)x + \frac{1}{2}\log\left(\frac{80}{80-x}\right)$

$= \log 2^x + \log\left(\frac{80}{80-x}\right)^{1/2}$

(1) $S_0(x) = e^{-\int_0^x \mu_z dz} = 2^{-x}\left(\frac{80}{80-x}\right)^{-1/2} = \left(\frac{1}{2}\right)^x \left(\frac{80-x}{80}\right)^{1/2}, \quad 0 \le x < 80$

$S_0(0) = 1$, $S_0(80) = 0$

clearly $\mu_x \ge 0$ and $S_0(x) \ge 0$ for all $x < 80$, then $-\mu_x S_0(x) \le 0 \Rightarrow S_0(x)$ is clearly non-increasing

$\frac{dS_0(x)}{dx} = -\mu_x S_0(x)$

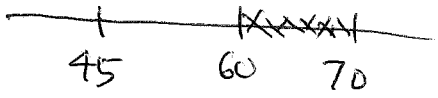
[1] Note that log in the expression is the natural logarithm.

(2) $${}_tp_x = \frac{S_0(x+t)}{S_0(x)} = \frac{\left(\frac{1}{2}\right)^{x+t}\left(\frac{80-x-t}{80}\right)^{1/2}}{\left(\frac{1}{2}\right)^{x}\left(\frac{80-x}{80}\right)^{1/2}}$$

$$= \left(\frac{1}{2}\right)^{t}\left(1-\frac{t}{80-x}\right)^{1/2}$$

This gives the probability that (x) survives another t years.

(3)



$${}_{15|10}q_{45} = {}_{45}p_{45} - {}_{25}p_{45}$$

$$= \left(\frac{1}{2}\right)^{15}\left(1-\frac{15}{35}\right)^{1/2} - \left(\frac{1}{2}\right)^{25}\left(1-\frac{25}{35}\right)^{1/2}$$

$$= .000023053\underline{19}$$