

MATH 3630 - Actuarial Mathematics I

Fall 2010 - Valdez

Homework No. 1

due ~~Wednesday~~, 6:15 PM, 13 September 2010

Monday

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For a certain population, the force of mortality for a newborn is given by

$$\mu_x = \frac{0.5}{x+1}, \quad \text{for } x > 0.$$

1. Derive the corresponding survival function $S_X(x)$ and demonstrate that it satisfies the important properties of a legitimate survival function.
2. Calculate the probability that a life aged 2 will survive another 3 years.
3. Calculate ${}_{2|2}q_1$ and interpret this probability.

Note that $\int_0^x \frac{0.5}{z+1} dz = 0.5 \log(x+1)$

$$\textcircled{1} S_X(x) = e^{-\int_0^x \mu_z dz} = e^{-0.5 \log(x+1)} = (x+1)^{-0.5}$$

$$\text{i) } \frac{d}{dx} S_X(x) = -0.5(x+1)^{-1.5} < 0 \Rightarrow \text{decreasing}$$

$$\text{ii) } S_X(0) = 1 \quad \text{and} \quad \lim_{x \rightarrow \infty} S_X(x) = S_X(\infty) = 0$$

$$\textcircled{2} P(T_2 > 3) = S_{T_2}(3) = S_X(5) / S_X(2) = (3/6)^{0.5} = \underline{\underline{0.7071}}$$

$$\textcircled{3} {}_{2|2}q_1 = \frac{S_X(3) - S_X(5)}{S_X(2)} = \frac{4^{-0.5} - 6^{-0.5}}{2^{-0.5}} = \underline{\underline{0.1298}}$$

prob that a life (1) will die between ages 3 and 5.

~~1~~ ~~3~~ ~~5~~