MATH 3630 - Actuarial Mathematics I
Fall 2009 - Valdez
Homework No. 1
due Wednesday, 6:50 PM, 16 September 2009

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In a certain population where 1/3 are females and 2/3 are males, the force of mortality for a female newborn is given by

$$\mu_x^{\mathrm{F}} = \frac{1}{2}x \quad \text{for } x \geq 0,$$

and that for a male newborn is

$$\mu_x^{\mathrm{M}} = \frac{8}{9}x \quad \text{for } x \geq 0.$$

For a randomly selected member of this population, calculate ${}_{1|2}q_3$ and interpret this probability.

Female $\Rightarrow$ $S_x^F(x) = e^{-\int_0^x \frac{1}{2}z\,dz} = e^{-\frac{1}{4}x^2}$

Male $\Rightarrow$ $S_x^M(x) = e^{-\int_0^x \frac{8}{9}z\,dz} = e^{-\frac{4}{9}x^2}$

Female $\Rightarrow$ ${}_{1|2}q_3^F = \dfrac{S_x^F(4) - S_x^F(6)}{S_x^F(3)} = \dfrac{e^{-4} - e^{-9}}{e^{-2.25}} = 0.1726$

Male $\Rightarrow$ ${}_{1|2}q_3^M = \dfrac{e^{-7.1111} - e^{-16}}{e^{-4}} = 0.0445$

For a randomly selected member,

${}_{1|2}q_3 = \frac{1}{3}(0.1726) + \frac{2}{3}(0.0445) = 0.0872$

This gives the probability that a randomly selected member who is age 3 will die between the ages 4 and 6.