MATH 3630 - Actuarial Mathematics I
Fall 2008 - Valdez
Homework No. 1
due Monday, 6:50 PM, September 8, 2008

Please return this page with your signature. Please write your name and student number at the spaces provided:

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Let X be the lifetime (of a newborn) random variable with SDF defined by

$$S_X(x) = e^{-(x/\lambda)^k} \text{for } x \geq 0,$$

where λ and k are both parameters.

1. Give constraints on the values of the parameters λ and k so that the function above is a legitimate SDF. Justify your solution.
2. Find the hazard rate at age x, μ_x.
3. Find an expression for $\mathring{e}_0$, the average future lifetime of a newborn.
4. Suppose $\lambda = \frac{15}{2}$ and $k = \frac{3}{4}$. Calculate ${}_{20|10}q_{20}$ and interpret this probability.

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$$S_X(x) = e^{-(x/\lambda)^k}, \quad x \geq 0$$

① For this to be a legit SDF, we must check:

i) non-increasing $\quad \frac{d}{dx} S_X(x) = -\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}$

because $x \geq 0$, Clearly $\lambda > 0$

and for $\frac{d}{dx} S_X(x) \leq 0 \Rightarrow k \geq 0$

Clearly $k=0$ leads to meaningless SDF

$\therefore \lambda > 0, \; k > 0$

ii) $S_X(0) = 1$ for any λ, k

iii) $S_X(\infty) = 0$ provided $\lambda > 0$ and $k > 0$.

② $$\mu_x = -\frac{d}{dx} \log S_X(x) = \frac{d}{dx}\left(\frac{x}{\lambda}\right)^k = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}.$$

③ $$\mathring{e}_0 = E(x) = \int_0^\infty S_X(x)\, dx$$

$$= \int_0^\infty e^{-\left(\frac{x}{\lambda}\right)^k} dx$$

Apply change of variable $\quad t = \left(\frac{x}{\lambda}\right)^k$ ✓

$$dt = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} dx$$

so that $dx = \frac{\lambda}{k} t^{(1/k)-1} dt$. Thus,

$$\mathring{e}_0 = \int_0^\infty \frac{\lambda}{k} t^{(1/k)-1} e^{-t} dt = \frac{\lambda}{k}\Gamma\left(\frac{1}{k}\right) = \lambda\,\Gamma\left(1+\frac{1}{k}\right)$$

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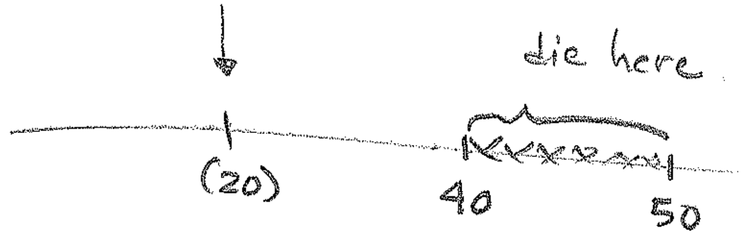
${}_{20|10}q_{20}$ is the probability that a life (20) will survive for 20 years but die within the next 10 years after that



$$= \frac{S_x(40) - S_x(50)}{S_x(20)} \qquad \lambda = 15/2 \quad k = 3/4$$

$$= \frac{e^{-(80/15)^{3/4}} - e^{-(100/15)^{3/4}}}{e^{-(40/15)^{3/4}}}$$

$$= \underline{\underline{0.1138642}}$$