Exercise 6.5

Let G the required gross annual premium and simply denote by $K = K_{[35]}$ the curtate future lifetime of a select age 35.

(a) The gross future loss random variable can be expressed as

$$L_0^g = \text{PVFB}_0 + \text{PVFE}_0 - \text{PVFP}_0,$$

where

$$\begin{aligned} \text{PVFB}_0 &= 100000v^{\min(K+1,20)} \\ \text{PVFE}_0 &= 3000 + 0.17G + 0.03G\ddot{a}_{\overline{\min(K+1,20)}|} \\ \text{PVFP}_0 &= G\ddot{a}_{\overline{\min(K+1,20)}|} \end{aligned}$$

so that

$$L_0^g = 100000v^{\min(K+1,20)} + 3000 + 0.17G - 0.97G\ddot{a}_{\overline{\min(K+1,20)}|}$$

* corrected on Feb 11, 2012 - thanks to N. Ioanna

(b) We set $\text{E}[L_0^g] = 0$ which gives us

$$\big(0.97\ddot{a}_{[35]:\overline{20}|} - 0.17\big)G = 100000A_{[35]:\overline{20}|} + 3000$$

so that

$$G = \frac{100000\big[1 - (1-v)\ddot{a}_{[35]:\overline{20}|}\big] + 3000}{0.97\ddot{a}_{[35]:\overline{20}|} - 0.17}$$

From the table for Standard Select Survival Model at 5%, we have

$$\begin{aligned} \ddot{a}_{[35]:\overline{20}|} &= \ddot{a}_{[35]} - v^{20}\frac{\ell_{55}}{\ell_{[35]}}\ddot{a}_{55} \\ &= 18.97415 - (1.05)^{-20} \cdot \frac{97846.20}{99549.01} \cdot 13.02489 \\ &= 13.02489 \end{aligned}$$

Therefore, we have

$$G = \frac{100000\big[1 - (1 - (1/1.05))13.02489\big] + 3000}{0.97(13.02489) - 0.17} = 3287.569$$

(c) Rewrite the gross future loss random variable as

$$\begin{aligned} L_0^g &= 100000v^{\min(K+1,20)} + 3000 + 0.17G - 0.97G\frac{1 - v^{\min(K+1,20)}}{d} \\ &= \left(100000 + \frac{0.97G}{d}\right) v^{\min(K+1,20)} + 3000 + 0.17G - \frac{0.97G}{d} \end{aligned}$$

The variance can thus be written as

$$\text{Var}[L_0^g] = \left(100000 + \frac{0.97G}{d}\right)^2 \left[{}^2A_{[35]:\overline{20}|} - \left(A_{[35]:\overline{20}|}\right)^2\right]$$

From the table for Standard Select Survival Model at 5%, we find

$$A_{[35]:\overline{20}|} = 1 - (1-v)\ddot{a}_{[35]:\overline{20}|} = 1 - (1 - (1/1.05))(13.02489) = 0.3797671$$

and

$$\begin{aligned} {}^2A_{[35]:\overline{20}|} &= {}^2A_{[35]} - v^{40}\,\frac{\ell_{55}}{\ell_{[35]}}\,{}^2A_{55} + v^{40}\,\frac{\ell_{55}}{\ell_{[35]}} \\ &= 0.01594 - (1.05)^{-40}\,\frac{97846.20}{99549.01}\,0.07483 + (1.05)^{-40}\,\frac{97846.20}{99549.01} \\ &= 0.1451085. \end{aligned}$$

Therefore, we find the standard deviation of the gross future loss random variable as

$$\text{SD}[L_0^g] = \left(100000 + \frac{0.97 \cdot 3287.569}{1 - (1/1.05)}\right) \cdot \sqrt{0.1451085 - (0.3797671)^2} = 4968.362.$$

(d) The event $L_0^g < 0$ is equivalent to the event

$$\left[100000 + (0.97G/d)\right]v^{\min(K+1,20)} < (0.97G/d) - 3000 - 0.17G$$

which implies

$$v^{\min(K+1,20)} < \frac{(0.97G/d) - 3000 - 0.17G}{100000 + (0.97G/d)}$$

or equivalently

$$K + 1 > -\frac{1}{\delta}\log\left[\frac{(0.97G/d) - 3000 - 0.17G}{100000 + (0.97G/d)}\right] = 19.84410.$$

Finally, we have

$$\begin{aligned} \Pr[L_0^g < 0] &= \Pr[K > 18.84410] = \Pr[K \geq 19] \\ &= {}_{19}p_{[35]} = \frac{\ell_{54}}{\ell_{[35]}} = \frac{98022.38}{99549.01} = 0.9846645. \end{aligned}$$

The contract makes a profit only if the person select age 35 will survive another 19 years.