Exercise 6.20

(a) Let P be the required annual premium. By the equivalence principle, we have

$$P = 250000\,\frac{A_{[40]:\overline{20}|}}{\ddot{a}_{[40]:\overline{20}|}}$$

where

$$\begin{aligned}\ddot{a}_{[40]:\overline{20}|} &= \ddot{a}_{[40]} - {}_{20}E_{[40]}\,\ddot{a}_{60} = \ddot{a}_{[40]} - v^{20}\,\frac{\ell_{60}}{\ell_{[40]}}\,\ddot{a}_{60} \\ &= 18.45956 - (1.05)^{-20}\,\frac{96634.14}{99327.82}\,14.90407 = 12.99471\end{aligned}$$

and

$$A_{[40]:\overline{20}|} = 1 - (1-(1/1.05))(12.99471) = 0.3812045,$$

giving us

$$P = 250000\,\frac{0.3812045}{12.99471} = 7333.842.$$

(b) Based on the equivalence principle, $\mathrm{E}[L_0] = 0$ and since, in this case, we have

$$L_0 = \left(250000 + \frac{P}{d}\right) v^{\min(K+1,20)} - \frac{P}{d},$$

then

$$\mathrm{Var}\big[L_0\big] = \left(250000 + \frac{P}{d}\right)^2 \mathrm{Var}\big[v^{\min(K+1,20)}\big] = \left(250000 + \frac{P}{d}\right)^2 \left[{}^2A_{[40]:\overline{20}|} - \left(A_{[40]:\overline{20}|}\right)^2\right],$$

where

$$\begin{aligned}{}^2A_{[40]:\overline{20}|} &= {}^2A_{[40]} - v^{40}\,\frac{\ell_{60}}{\ell_{[40]}}\,{}^2A_{60} + v^{40}\,\frac{\ell_{60}}{\ell_{[40]}} \\ &= 0.02338 - (1.05)^{-40}\frac{96634.14}{99327.82}(0.10834) + (1.05)^{-40}\frac{96634.14}{99327.82} \\ &= 0.1466016\end{aligned}$$

so that the standard deviation of L_0 is

$$\begin{aligned}\mathrm{SD}\big[L_0\big] &= \left(250000 + \frac{7333.842}{1-(1/1.05)}\right)\sqrt{0.1466016 - (0.3812045)^2} \\ &= 14481.31.\end{aligned}$$

The answer is off a bit from the textbook answer.

(c) For 10,000 identical, independent contracts, the 99th percentile of the (aggregate) net future loss is given by

$$z_{0.99} \times \sqrt{10000} \times \mathrm{SD}\big[L_0\big] = 2.326(100)(14481.31) = 3368353,$$

based on the Normal approximation with $z_{0.99}$ denoting the 99th percentile of a standard Normal. The answer does not match the textbook answer.