Exercise 6.16

For a whole life insurance of 150000 to (x), the net future loss random variable is

$$\begin{aligned} L_0^n &= 150000 v^{K+1} - P\,\ddot{a}_{\overline{K+1}|} \\ &= \left(150000 + \frac{P}{d}\right) v^{K+1} - \frac{P}{d}, \end{aligned}$$

where, based on the equivalence principle,

$$P = 150000 \frac{A_x}{\ddot{a}_x}.$$

The variance of this future loss can be expressed as

$$\begin{aligned} \mathrm{Var}\big[L_0^n\big] &= \left(150000 + \frac{P}{d}\right)^2 \mathrm{Var}\big[v^{K+1}\big] \\ &= \left(150000 + \frac{P}{d}\right)^2 \left[{}^2A_x - (A_x)^2\right] \\ &= (150000)^2 \left(1 + \frac{A_x}{d\ddot{a}_x}\right)^2 \left[{}^2A_x - (A_x)^2\right] \\ &\quad \text{since } A_x + d\ddot{a}_x = 1 \\ &= (150000)^2 \left(\frac{1}{1 - A_x}\right)^2 \left[{}^2A_x - (A_x)^2\right] \end{aligned}$$

The standard deviation of this future loss is therefore

$$\mathrm{SD}\big[L_0^n\big] = (150000)\left(\frac{1}{1 - 0.0653}\right) \sqrt{[0.0143 - (0.0653)^2]} = 16076.72.$$