Exercise 6.13

First consider the net annual premium. Denote this by P. Assuming UDD between integral ages (not clearly specified in the problem), the APV of the benefits is

$$\begin{aligned}
\text{APV(benefits)} &= 10000\left(10\bar{A}_{[40]} - 8\,{}_{20}E_{[40]}\,\bar{A}_{60}\right) \\
&= 10000\frac{i}{\delta}\left(10A_{[40]} - 8\,{}_{20}E_{[40]}\,A_{60}\right) \\
&= 10000\frac{0.06}{\log(1.06)}[10(0.2329971) - 8(0.2348948)(0.4568085)] \\
&= 15152.74
\end{aligned}$$

and the APV of the premiums is

$$\text{APV(premiums)} = P \cdot \ddot{a}_{[40]:\overline{20}|} = 11.29624P$$

Therefore, we have $P = 15152.74/11.29624 = 1341.396$.

The following R code calculates the actuarial functions used in the calculations above:

```
A <- 0.0001
B <- 0.00035
c <- 1.075
surv <- function(x){
exp(-A*x-(B*(c^x-1)/log(c)))}
# last possible age is 165
x <- 40:165
p40k <- surv(x+1)/surv(x)
q40k <- 1-p40k
# replace p's and q's with select rates
q40k[1] <- 0.75*q40k[1]
q40k[2] <- 0.9*q40k[2]
p40k[1] <- 1-q40k[1]
p40k[2] <- 1-q40k[2]
kp40 <- cumprod(c(1,p40k[-length(x)]))
k <- 0:(length(x)-1)
int <- 0.06
v <- 1/(1+int)
delta <- log(1+int)
vk <- v^(k+1)
A40 <- sum(vk*kp40*q40k)
A40c <- (int/delta)*A40
E4020 <- v^20 * kp40[21]

x <- 60:165
p60k <- surv(x+1)/surv(x)
q60k <- 1-p60k
kp60 <- cumprod(c(1,p60k[-length(x)]))
k <- 0:(length(x)-1)
```

```
vk <- v^(k+1)
A60 <- sum(vk*kp60*q60k)
A60c <- (int/delta)*A60

A4020 <- A40 - E4020*A60 + E4020
a4020 <- (1-A4020)/(1-v)
num <- 10000*(10*A40c - 8*E4020*A60c)
den <- a4020
NP <- num/den
```

This produces the results:

```
> A40
[1] 0.2329971
> A60
[1] 0.4568085
> E4020
[1] 0.2348948
> a4020
[1] 11.29624
> NP
[1] 1341.396
```

For the gross premium calculation, we need the APV of the expenses:

$$\begin{aligned}\text{APV(expenses)} &= 0.27G + 0.03G\,\ddot{a}_{[40]:\overline{20|}} + 10\sum_{k=0}^{19}(1.03)^k v^k {}_kp_{[40]} \\ &= 0.27G + 0.03G\,\ddot{a}_{[40]:\overline{20|}} + 10\left(\ddot{a}_{[40]:\overline{20|}}\right)_{i^*},\end{aligned}$$

where $\left(\ddot{a}_{[40]:\overline{20|}}\right)_{i^*}$ is a 20-year temporary life annuity-due evaluated at interest rate $i^* = (1.06/1.03) - 1$. The R code for calculating this is given below.
It is easy to verify that the gross annual premium is therefore

$$\begin{aligned}G &= \frac{\text{APV(benefits)} + 10\left(\ddot{a}_{[40]:\overline{20|}}\right)_{i^*}}{0.97\,\ddot{a}_{[40]:\overline{20|}} - 0.27} \\ &= \frac{15152.74 + 10(14.17361)}{0.97(11.29624) - 0.27} \\ &= 1431.081\end{aligned}$$

```
x <- 40:165
p40k <- surv(x+1)/surv(x)
q40k <- 1-p40k
# replace p's and q's with select rates
q40k[1] <- 0.75*q40k[1]
q40k[2] <- 0.9*q40k[2]
p40k[1] <- 1-q40k[1]
```

```
p40k[2] <- 1-q40k[2]
kp40 <- cumprod(c(1,p40k[-length(x)]))
k <- 0:19
kp40 <- kp40[1:20]
int <- (1.06/1.03)-1
v <- 1/(1+int)
vk <- v^k
a4020s <- sum(vk*kp40)
```

This produces the result:

```
> a4020s
[1] 14.17361
```