Exercise 6.11

Denote by P the single benefit premium for the annuity.

(a) The APV of the benefits is the single benefit premium:

$$P = 20000\,{}_{30}E_{[35]}\,\ddot{a}_{65} + P\,A^{\;\;1}_{[35]:\overline{30}|}$$

Solving for P, we get

$$\begin{aligned} P &= \frac{20000\,{}_{30}E_{[35]}\,\ddot{a}_{65}}{1 - A^{\;\;1}_{[35]:\overline{30}|}} \\ &= \frac{20000(0.2198276)(13.54979)}{1-0.01848040} = 60694, \end{aligned}$$

where we can verify that

$$\begin{aligned} {}_{30}E_{[35]} &= v^{30}\frac{\ell_{65}}{\ell_{[35]}} = (1.05)^{-30}\frac{94579.73}{99549.01} = 0.2198276 \\ \ddot{a}_{[35]:\overline{30}|} &= \ddot{a}_{[35]} - {}_{30}E_{[35]}\,\ddot{a}_{65} \\ &= 18.97415 - 0.2198276(13.54979) = 15.99553 \\ A^{\;\;1}_{[35]:\overline{30}|} &= A_{[35]:\overline{30}|} - {}_{30}E_{[35]} \\ &= 1 - (1-(1/1.05))(15.99553) - 0.2198276 = 0.01848040 \end{aligned}$$

(b) Total annuity payments therefore will not exceed P, the single benefit premium, if death is within the first 3 years after turning 65. The APV of this benefit option therefore can be expressed as

$$\begin{aligned} \text{APV(option)} &= {}_{30}E_{[35]} \times \left(40694\,v\,q_{65} + 20694\,v^2\,p_{65}\,q_{66} + 694\,v^3\,{}_2p_{65}\,q_{67}\right) \\ &= {}_{30}E_{[35]} \times \frac{v}{\ell_{65}}\left(40694\,d_{65} + 20694\,v\,d_{66} + 694\,v^2\,d_{67}\right) \\ &= (0.2198276) \times \frac{(1.05)^{-1}}{94579.73}\big(40694 \cdot 559.40 + 20694(1.05)^{-1}\cdot 622.28 \\ &\qquad + 694(1.05)^{-2}\cdot 691.99\big) \\ &= 78.50247 \end{aligned}$$

Thus, the revised premium is P + APV(option) = 60694 + 78.50247 = 60772.50247. Answer slightly different from the textbook.