Exercise 5.18

Choose $f(T) = \bar{a}_{\overline{T}|}$ and note that

$$f(t) = \frac{1 - v^t}{\delta}$$

from which it follows that

$$f'(t) = v^t > 0 \quad \text{and} \quad f''(t) = -\delta v^t < 0.$$

Henceforth, by Jensen's inequality, we have

$$\mathrm{E}\big[\bar{a}_{\overline{T}|}\big] = \bar{a}_x \leq \bar{a}_{\overline{\mathrm{E}[T]}|}.$$

Also, because $v^t = 1 - \delta \bar{a}_{\overline{t}|}$, we have

$$\mathrm{E}[v^T] = A_x = 1 - \delta \bar{a}_x \geq 1 - \delta \bar{a}_{\overline{\mathrm{E}[T]}|} = v^{\mathrm{E}[T]}.$$

To interpret these results, a life annuity is always cheaper than an annuity-certain for a period of the expected future lifetime; a life insurance is always more expensive than a discounted value of a dollar paid on the expected future lifetime.