Exercise 5.16

(a) For the arithmetically increasing 10-year life annuity-due on (50), we can write the present value random variable of the benefits as

$$Y = (I\ddot{a})_{\overline{K+1|}} I(K \le 9) + (I\ddot{a})_{\overline{10|}} I(K > 9),$$

where K is the curtate future lifetime of (50) with probability mass

$$\Pr[K = k] = {}_kp_{50}\, q_{50+k}.$$

The following R code calculates the expected value and variance of Y based on the Standard Ultimate Survival Model with $i = 5\%$:

```
# arithmetically increasing 1,2,...,10
A <- .00022
B <- 2.7*10^(-6)
c <- 1.124
surv <- function(x){
exp(-A*x-(B*(c^x-1)/log(c)))}
x <- 50:59
px <- surv(x+1)/surv(x)
qx <- 1-px
int <- .05
v <- 1/(1+int)
vcum <- v^(0:(length(x)-1))
bena <- 1:10
# check if consistent with the current payment technique
# EY <- sum(bena*vcum*cumprod(c(1,px[-length(px)])))
y <- cumsum(bena*vcum)
prob <- cumprod(c(1,px[-length(px)]))*qx
prob[length(prob)] <- 1 - sum(prob[-length(prob)])
EY <- sum(y*prob)
EY2 <- sum(y^2 * prob)
VarY <- EY2 - EY^2
```

This produces the results:

```
> EY
[1] 40.95364
> VarY
[1] 11.0571
```

(b) For the geometrically increasing 10-year life annuity-due on (50), first we note that the present value random variable for $K = 0, 1, \ldots, 9$

$$Y = \sum_{j=0}^{K} \frac{(1.03)^j}{(1.05)^{j+1}} = \frac{1}{1.03} \sum_{j=0}^{K} \frac{1}{(1.05/1.03)^{j+1}} = \frac{1}{1.03} \ddot{a}^*_{\overline{K+1|}},$$

where $\ddot{a}^{*}_{\overline{K+1}|}$ is an annuity-due evaluated at interest rate $(1.05/1.03) - 1$. Therefore,

$$Y = 11.03\ddot{a}^{*}_{\overline{K+1}|} I(K \leq 9) + 11.03\ddot{a}^{*}_{\overline{10}|} I(K \geq 10),$$

where K is the curtate future lifetime of (50) with probability mass

$$\Pr[K = k] = {}_{k}p_{50}\, q_{50+k}.$$

The following R code calculates the expected value and variance of Y based on the Standard Ultimate Survival Model with $i = 5\%$:

```
# geometrically increasing 1, 1.03, 1.03^2, ...
beng <- (1.03)^(0:(length(x)-1))
# check if consistent with the current payment technique
# EY <- sum(beng*vcum*cumprod(c(1,px[-length(px)])))
y <- cumsum(beng*vcum)
EY <- sum(y*prob)
EY2 <- sum(y^2 * prob)
VarY <- EY2 - EY^2
```

This produces the results:

```
> EY
[1] 9.121096
> VarY
[1] 0.3296498
```