Exercise 4.22

(a) Starting with $t = 1/40$, we evaluate ${}_tp_{60}$ using

$${}_tp_{60} = \exp -\left[\int_0^t \mu_{60+s} ds\right] = \exp -\left[\sum_{j=0}^{40t-1} \int_{j/40}^{(j+1)/40} \mu_{60+s} ds\right],$$

where we use repeated Simpson's rule with $h = 1/80$ to approximate the integral

$$\int_{j/40}^{(j+1)/40} \mu_{60+s} ds \approx \frac{h}{3}\left[\mu_{60+j/40} + 4\mu_{60+h+j/40} + \mu_{60+(j+1)/40}\right].$$

The following R code produces a table of approximate values for ${}_tp_{60}$:

```
A <- 3.5*10^(-4)
B <- 5.5*10^(-4)
C <- 1.00085
D <- 1.0005
# function to evaluate force of mortality mu(60+s)
mu60 <- function(s){
A + B*(C^(60+s))*(D^((60+s)^2))}
t <- seq(0,2,1/40)
tp60 <- rep(0,length(t))
tp60[1] <- 1
n <- 1
temp <- 0
h <- 1/80
while (n<length(t)) {
n <- n+1
temp <- temp + (h/3)*(mu60(t[n-1]) + 4*mu60(t[n-1]+h) + mu60(t[n]))
tp60[n] <- exp(-temp)
}
t_out <- c(t,NA)
tp60_out <- c(tp60,NA)
output <- cbind(t_out[1:41],tp60_out[1:41],t_out[42:82],tp60_out[42:82])
colnames(output) <- c("t","tp60","t","tp60")
print(output)
```

This gives the output

```
> print(output)
          t      tp60     t      tp60
 [1,] 0.000 1.0000000 1.025 0.9959474
 [2,] 0.025 0.9999037 1.050 0.9958458
 [3,] 0.050 0.9998073 1.075 0.9957441
 [4,] 0.075 0.9997107 1.100 0.9956423
```

```
 [5,] 0.100 0.9996140 1.125 0.9955403
 [6,] 0.125 0.9995172 1.150 0.9954382
 [7,] 0.150 0.9994203 1.175 0.9953359
 [8,] 0.175 0.9993232 1.200 0.9952335
 [9,] 0.200 0.9992261 1.225 0.9951310
[10,] 0.225 0.9991288 1.250 0.9950284
[11,] 0.250 0.9990313 1.275 0.9949256
[12,] 0.275 0.9989338 1.300 0.9948227
[13,] 0.300 0.9988361 1.325 0.9947196
[14,] 0.325 0.9987383 1.350 0.9946164
[15,] 0.350 0.9986404 1.375 0.9945131
[16,] 0.375 0.9985423 1.400 0.9944096
[17,] 0.400 0.9984441 1.425 0.9943060
[18,] 0.425 0.9983458 1.450 0.9942023
[19,] 0.450 0.9982473 1.475 0.9940984
[20,] 0.475 0.9981488 1.500 0.9939944
[21,] 0.500 0.9980501 1.525 0.9938902
[22,] 0.525 0.9979512 1.550 0.9937860
[23,] 0.550 0.9978523 1.575 0.9936815
[24,] 0.575 0.9977532 1.600 0.9935770
[25,] 0.600 0.9976540 1.625 0.9934723
[26,] 0.625 0.9975546 1.650 0.9933674
[27,] 0.650 0.9974552 1.675 0.9932624
[28,] 0.675 0.9973556 1.700 0.9931573
[29,] 0.700 0.9972558 1.725 0.9930520
[30,] 0.725 0.9971560 1.750 0.9929466
[31,] 0.750 0.9970560 1.775 0.9928411
[32,] 0.775 0.9969559 1.800 0.9927354
[33,] 0.800 0.9968556 1.825 0.9926296
[34,] 0.825 0.9967552 1.850 0.9925236
[35,] 0.850 0.9966547 1.875 0.9924175
[36,] 0.875 0.9965540 1.900 0.9923113
[37,] 0.900 0.9964533 1.925 0.9922049
[38,] 0.925 0.9963524 1.950 0.9920984
[39,] 0.950 0.9962513 1.975 0.9919917
[40,] 0.975 0.9961501 2.000 0.9918849
[41,] 1.000 0.9960488    NA        NA
```

From this table, we find that

$$
\begin{aligned}
{}_{1/4}p_{60} &= 0.9990313 \\
p_{60} &= 0.9960488 \\
{}_{2}p_{60} &= 0.9918849 \quad \text{does not match textbook answer}
\end{aligned}
$$

(b) To approximate the 2-year term insurance issued to (60), we use the approximate values obtained from (a) as follows:

$$A^{1}_{60:\overline{2}|} = \int_0^2 v^t {}_tp_{60}\mu_{60+t}dt = \sum_{j=0}^{78} \int_{j/40}^{(j+2)/40} v^t {}_tp_{60}\mu_{60+t}dt,$$

where we use repeated Simpson's rule with $h = 1/40$ to approximate the integral

$$\int_{j/40}^{(j+2)/40} v^t {}_tp_{60}\mu_{60+t}dt \approx \frac{h}{3}\left[v^{j/40} {}_{j/40}p_{60}\mu_{60+j/40} + 4v^{(j+1)/40} {}_{(j+1)/40}p_{60}\mu_{60+(j+1)/40} + v^{(j+2)/40} {}_{(j+2)/40}p_{60}\mu_{60+(j+2)/40}\right].$$

The following R code calculates this approximation:

```
n <- 1
A60term2 <- 0
int <- 0.05
v <- 1/(1+int)
while (n<length(t)) {
n <- n+2
A60term2 <- A60term2 + (2*h/3)*(v^(t[n-2])*tp60[n-2]*mu60(t[n-2]) +
   4*v^(t[n-1])*tp60[n-1]*mu60(t[n-1]) + v^(t[n])*tp60[n]*mu60(t[n]))
}
A60term2
```

and produces the following result:

```
> A60term2
[1] 0.007725168
```