

**Exercise 4.15**

Although **not clearly stated in the problem**, we will assume that benefits are payable at the moment of death. Begin by writing  $Z_1 = \begin{cases} v^T, & T \leq n \\ 0, & T > n \end{cases} = v^T I(T \leq n)$  and  $Z_2 = v^T$ , from which it follows that  $Z_1 Z_2 = v^{2T} I(T \leq n)$  and

$$E[Z_1 Z_2] = E[v^{2T} I(T \leq n)] = {}^2\bar{A}_{x:\overline{n}|}^1.$$

The required covariance therefore can be written as

$$\text{Cov}[Z_1, Z_2] = E[Z_1 Z_2] - E[Z_1]E[Z_2] = {}^2\bar{A}_{x:\overline{n}|}^1 - \bar{A}_{x:\overline{n}|}^1 \bar{A}_x.$$