Exercise 4.11

Consider any $i^* > i$ and let $v = 1/(1+i)$ and $v_* = 1/(1+i^*)$. Then clearly $v > v_*$ so that

$$\begin{aligned}(\bar{A}_x)_i &= \int_0^\infty v^t f_x(t)dt \\ &> \int_0^\infty v_*^t f_x(t)dt = (\bar{A}_x)_{i^*}\end{aligned}$$

The result immediately follows. Another way to prove this is to take the derivative of $\bar{A}_x$ with respect to i. First note that

$$\frac{d}{di}v^t = -t \cdot v^t \cdot \frac{d\delta}{di} = -t \cdot v^t \cdot v = -t \cdot v^{t+1}.$$

It follows therefore that

$$\frac{d}{di}\bar{A}_x = \frac{d}{di}\int_0^\infty v^t f_x(t)dt = -\int_0^\infty t v^{t+1} f_x(t)dt < 0.$$

For larger interest rate, the time value of money leads to smaller present values and thus smaller actuarial present values.