Exercise 3.5

(a) We have

$$\begin{aligned}
{}_7p_{[70]} &= p_{[70]} \cdot p_{[70]+1} \cdot p_{[70]+2} \cdot p_{[70]+3} \cdot p_{[70]+4} \cdot p_{75} \cdot p_{76} \\
&= (1-q_{[70]})(1-q_{[70]+1})(1-q_{[70]+2})(1-q_{[70]+3})(1-q_{[70]+4})(1-q_{75})(1-q_{76})) \\
&= (1-0.010373)(1-0.014330)(1-0.019192)(1-0.025023)(1-0.031859) \\
&\qquad \times (1-0.043686)(1-0.048270) \\
&= 0.821929.
\end{aligned}$$

(b) We have

$$\begin{aligned}
{}_{1|2}q_{[70]+2} &= p_{[70]+2} \cdot {}_2q_{[70]+3} = p_{[70]+2} \cdot (1 - {}_2p_{[70]+3}) \\
&= p_{[70]+2} \cdot (1 - p_{[70]+3} \cdot p_{[70]+4}) \\
&= p_{[70]+2} \cdot [1 - (1-q_{[70]+3})(1-q_{[70]+4})] \\
&= (1-0.019192)[1-(1-0.025023)(1-0.031859)] \\
&= 0.05500841.
\end{aligned}$$

(c) First note that we have

$${}_{3.8}q_{[70]+0.2} = {}_{0.8}q_{[70]+0.2} + {}_{0.8}p_{[70]+0.2} \cdot {}_3q_{[70]+1},$$

where ${}_{0.8}q_{[70]+0.2}$ can de derived from the equation

$$q_{[70]} = {}_{0.2}q_{[70]} + {}_{0.2}p_{[70]} \cdot {}_{0.8}q_{[70]+0.2}.$$

Assuming UDD within integral ages, we have

$${}_{0.8}q_{[70]+0.2} = \frac{q_{[70]} - {}_{0.2}q_{[70]}}{{}_{0.2}p_{[70]}} = \frac{0.8\, q_{[70]}}{1 - 0.2\, q_{[70]}} = \frac{0.8(0.010373)}{1-0.2(0.010373)} = 0.008315652.$$

Subsequently, we have

$$\begin{aligned}
{}_{3.8}q_{[70]+0.2} &= {}_{0.8}q_{[70]+0.2} + (1 - {}_{0.8}q_{[70]+0.2})(1 - {}_3p_{[70]+1}) \\
&= 0.008315652 + (1-0.008315652) \\
&\qquad \times [1 - (1 - (1-0.014330)(1-0.019192)(1-0.025023))] \\
&= 0.06527606.
\end{aligned}$$