Exercise 3.2

When ℓ_x's are given, it is better to use the direct linear interpolation formula for UDD

$$\ell_{x+t} = (1-t)\ell_x + t\ell_{x+1}$$

and the direct exponential interpolation formula for constant force

$$\ell_{x+t} = \ell_x^{1-t} \cdot \ell_{x+1}^{t}.$$

(a) Assuming UDD, we have

$$\begin{aligned} {}_{0.2}q_{52.4} &= 1 - {}_{0.2}p_{52.4} = 1 - \frac{\ell_{52.6}}{\ell_{52.4}} = 1 - \frac{0.4\ell_{52} + 0.6\ell_{53}}{0.6\ell_{52} + 0.4\ell_{53}} \\ &= 1 - \frac{0.4(89948) + 0.6(89089)}{0.6(89948) + 0.4(89089)} = 1 - \frac{89432.6}{89604.4} = 0.001917317. \end{aligned}$$

(b) Assuming constant force, we have

$${}_{0.2}q_{52.4} = 1 - \frac{\ell_{52}^{0.4} \cdot \ell_{53}^{0.6}}{\ell_{52}^{0.6} \cdot \ell_{53}^{0.4}} = 1 - \left(\frac{\ell_{53}}{\ell_{52}}\right)^{0.2} = 1 - \left(\frac{89089}{89948}\right)^{0.2} = 0.001917331.$$

(c) Assuming UDD, we have

$${}_{5.7}p_{52.4} = \frac{\ell_{58.1}}{\ell_{52.4}} = \frac{0.9\ell_{58} + 0.1\ell_{59}}{89604.4} = \frac{0.9(83940) + 0.1(82719)}{89604.4} = \frac{83817.9}{89604.4} = 0.9354217.$$

(d) Assuming constant force, we have

$${}_{5.7}p_{52.4} = \frac{\ell_{58.1}}{\ell_{52.4}} = \frac{\ell_{58}^{0.9} \cdot \ell_{59}^{0.1}}{\ell_{52}^{0.6} \cdot \ell_{53}^{0.4}} = \frac{83940^{0.9} \cdot 82719^{0.1}}{89948^{0.6} \cdot 89089^{0.4}} = \frac{83817.1}{89603.41} = 0.9354230.$$

(e) Assuming UDD, we have

$${}_{3.2|2.5}q_{52.4} = \frac{\ell_{55.6} - \ell_{58.1}}{\ell_{52.4}} = \frac{[0.4(87208) + 0.6(86181)] - 83817.9}{89604.4} = \frac{2773.9}{89604.4} = 0.03095719.$$

(f) Assuming constant force, we have

$${}_{3.2|2.5}q_{52.4} = \frac{\ell_{55.6} - \ell_{58.1}}{\ell_{52.4}} = \frac{87208^{0.4} \cdot 86181^{0.6} - 83817.1}{89603.41} = \frac{2773.247}{89603.41} = 0.03095024.$$