Exercise 2.5

Clearly, $F_0(t)$ is the cdf of an Exponential with mean $1/\lambda$. So T_0 has an Exponential distribution.

(a) Since $S_0(t) = e^{-\lambda t}$, we have

$$S_x(t) = \frac{S_0(x+t)}{S_0(x)} = \frac{e^{-\lambda(x+t)}}{e^{-\lambda x}} = e^{-\lambda t}.$$

Thus, we see that T_x also has the same Exponential distribution as T_0.

(b) $\mu_x = \dfrac{-dS_0(x)/dx}{S_0(x)} = \dfrac{\lambda e^{-\lambda x}}{e^{-\lambda x}} = \lambda$, which is independent of x and therefore is said to have a constant force of mortality for all x.

(c) The curtate expectation of life for a person age x can be derived as

$$e_x = \sum_{k=1}^{\infty} {}_kp_x = \sum_{k=1}^{\infty} S_x(k) = \sum_{k=1}^{\infty} e^{-\lambda k} = \frac{e^{-\lambda}}{1-e^{-\lambda}} = \frac{1}{e^{\lambda}-1},$$

which is independent of age x.

(d) For human mortality, force of mortality generally increases with age x especially as we become much older and average remaining future lifetime generally decreases with age (the older we get, sadly, closer we are to death). Neither of these characteristics is exhibited by the Exponential distribution.