

Exercise 2.16

Start with:

$$e_{x:\overline{n}|} = E[\min(K_x, n)] = \sum_{k=0}^{\infty} \min(k, n) {}_k|q_x = \sum_{k=1}^{n-1} k {}_k|q_x + n \sum_{k=n}^{\infty} {}_k|q_x$$

Now by noting that

$$\sum_{k=1}^{n-1} k {}_k|q_x = \sum_{k=1}^{n-1} k ({}_k p_x - {}_{k+1} p_x) = \sum_{k=1}^{n-1} k p_x - (n-1) {}_n p_x$$

and that

$$\sum_{k=n}^{\infty} {}_k|q_x = {}_n p_x,$$

we get the desired result:

$$e_{x:\overline{n}|} = \sum_{k=1}^{n-1} k p_x - (n-1) {}_n p_x + n {}_n p_x = \sum_{k=1}^n k p_x$$