Exercise 2.13

(a) We are given $\mu_x^* = 2\mu_x$ where $*$ refers to smokers and unstarred, non-smokers. It is easy to verify that

$$ {}_tp_x^* = \exp\left(-\int_0^t \mu_{x+s}^* ds\right) = \exp\left(-2\int_0^t \mu_{x+s} ds\right) = \left[\exp\left(-\int_0^t \mu_{x+s} ds\right)\right]^2 = \left({}_tp_x\right)^2. $$

Note that because ${}_tp_x \leq 1$, then ${}_tp_x^* = \left({}_tp_x\right)^2 \leq {}_tp_x$. Intuitively, survival of smokers are worse than non-smokers.

(b) The life expectancy for a 50-year-old non-smoker can be expressed as

$$ \mathring{e}_{50} = \int_0^\infty {}_tp_{50} dt, $$

where ${}_tp_{50} = \exp\left\{-\frac{B}{\log(c)} c^{50}(c^t - 1)\right\}$. On the other hand, the life expectancy for a 50-year-old smoker can be found using

$$ \mathring{e}_{50}^* = \int_0^\infty \left({}_tp_{50}\right)^2 dt, $$

The R code to evaluate the difference between these two life expectancies is given below (integrals are approximated using repeated Simpson's rule):

```
B <- 0.0005
c <- 1.07
tp50ns <- function (t) {
temp <- (B/log(c))*c^50*(c^t-1)
exp(-temp)}
tp50s <- function (t) {
temp <- tp50ns(t)
temp^2}
exc50.ns <- function(tol) {
a<-0
h<-.25
k<-0
v1 <- (h/3)*(tp50ns(a) + 4*tp50ns(a+h) + tp50ns(a+2*h))
v <- v1
while (v1 > tol) {
k <- k+2
lim1 <- a+k*h
mid <- a+(k+1)*h
lim2 <- a+(k+2)*h
v1 <- (h/3)*(tp50ns(lim1) + 4*tp50ns(mid) + tp50ns(lim2))
v <- v + v1}
v}
```

```
exc50.sm <- function(tol) {
a<-0
h<-.25
k<-0
v1 <- (h/3)*(tp50s(a) + 4*tp50s(a+h) + tp50s(a+2*h))
v <- v1
while (v1 > tol) {
k <- k+2
lim1 <- a+k*h
mid <- a+(k+1)*h
lim2 <- a+(k+2)*h
v1 <- (h/3)*(tp50s(lim1) + 4*tp50s(mid) + tp50s(lim2))
v <- v + v1}
v}
tol <- 10^(-50)
ec50ns <- exc50.ns(tol)
ec50sm <- exc50.sm(tol)
ec50ns
ec50sm
ec50ns-ec50sm
```

The output is given by

```
> ec50ns
[1] 21.20182
> ec50sm
[1] 14.76935
> ec50ns-ec50sm
[1] 6.432468
```

According to this result, for a 50-year-old, there is a difference of 6.4 extra years of life expectancy between that of a non-smoker and a smoker.

(c) To calculate the variances, we use

$$\mathrm{Var}[T_{50}] = \int_0^\infty t^2 {}_tp_{50}\mu_{50+t}dt - (\mathring{e}_{50})^2$$

and

$$\mathrm{Var}[T_{50}^*] = \int_0^\infty 2t^2 \left({}_tp_{50}\right)^2 \mu_{50+t}dt - (\mathring{e}_{50}^*)^2$$

where the integrals in each of the first term in the variance formula are approximated using repeated Simpson's rule. The following R code evaluates these respective variances:

```
f50sq.ns <- function (t) {
temp1 <- tp50ns(t)
temp2 <- B*c^(50+t)
temp3 <- t^2
```

```
temp1*temp2*temp3}
f50sq.sm <- function (t) {
temp1 <- tp50s(t)
temp2 <- 2*B*c^(50+t)
temp3 <- t^2
temp1*temp2*temp3}
esq50.ns <- function(tol) {
a<-0
h<-.25
k<-0
v1 <- (h/3)*(f50sq.ns(a) + 4*f50sq.ns(a+h) + f50sq.ns(a+2*h))
v <- v1
while (v1 > tol) {
k <- k+2
lim1 <- a+k*h
mid <- a+(k+1)*h
lim2 <- a+(k+2)*h
v1 <- (h/3)*(f50sq.ns(lim1) + 4*f50sq.ns(mid) + f50sq.ns(lim2))
v <- v + v1}
v}
esq50.sm <- function(tol) {
a<-0
h<-.25
k<-0
v1 <- (h/3)*(f50sq.sm(a) + 4*f50sq.sm(a+h) + f50sq.sm(a+2*h))
v <- v1
while (v1 > tol) {
k <- k+2
lim1 <- a+k*h
mid <- a+(k+1)*h
lim2 <- a+(k+2)*h
v1 <- (h/3)*(f50sq.sm(lim1) + 4*f50sq.sm(mid) + f50sq.sm(lim2))
v <- v + v1}
v}
tol <- 10^(-50)
var.ns <- esq50.ns(tol) - (ec50ns)^2
var.sm <- esq50.sm(tol) - (ec50sm)^2
var.ns
var.sm
```

The output:

```
> var.ns  (for non-smokers)
[1] 125.8860
> var.sm  (for smokers)
[1] 80.11494
```