MATH 3630

Actuarial Mathematics I

Final Examination

Tuesday, 11 December 2018

Time Allowed: 2 hours (1:00 - 3:00 pm)

Room: OAK 117

Total Marks: 120 points

Please write your name and student number at the spaces provided:

Name: ______________________________ **Student ID:** ______________________

- **There are twelve (12) written-answer questions here and you are to answer all twelve. Each question is worth 10 points. Your final mark will be divided by 120 to convert to a unit of 100%.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught cheating will be subject to university's disciplinary action.**
- **Good luck.**
- **Have a Happy and Healthy Christmas and New Year!**

Question	Worth	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total	120	
%	$\div$ 120	

Question No. 1:

The mortality pattern for a cohort of newborn can be described by

$$\mu_x = \begin{cases} 0.01, & \text{for } 0 < x \leq 40 \\ 0.04, & \text{for } x > 40 \end{cases}$$

A medical breakthrough reduces the force of mortality for age beyond 40 by 25%, but will not affect mortality prior to, and including, age 40.

Calculate the percentage improvement in the probability of a 25-year-old reaching to age 65 as a result of this medical breakthrough.

Question No. 2:

For a life (65), you are given the following extract from a life table:

k	ℓ_{65+k}
0	5,000
1	4,900
2	4,700
3	4,400
4	4,000
5	3,500

Mortality between integer ages is assumed to follow Uniform Distribution of Death (UDD).

Calculate the probability that (66) will die between ages 68.25 and 69.40.

Question No. 3:

You are given:

- For age prior to 45, mortality follows a constant force with $\mu = 0.02$.
- For ages 45 and later, mortality follows the `Survival Ultimate Life Table`.
- $i = 0.05$
- Z is the present value random variable for a whole life insurance of 1 payable at the end of the year of death issued to (40).

Calculate the probability that Z will be greater than 0.6.

Question No. 4:

You are given:

- The following extract from a mortality table:

x	95	96	97	98	99	100
ℓ_x	1000	750	-	200	50	0

- $v = 0.90$
- $\ddot{a}_{95} = 2.2$
- $\ddot{a}_{97} = 1.3$

Calculate ℓ_{97}.

Question No. 5:

A fully discrete whole life insurance of 1000 is issued to (46). You are given:

- Expenses consist of 10% of annual gross premium in the first year and 5% in subsequent years.
- $A_{45} = 0.15$
- $p_{45} = 0.99$
- $i = 0.04$

Calculate the gross annual premium for this policy.

Question No. 6:

For a special whole life insurance on (50), you are given:

- Death benefit, payable at the end of the year of death, consists of 1000 plus the return of all premiums without interest.
- Annual net premium of 16.95 is payable at the beginning of each year.
- $i = 0.05$
- $(IA)_{50} = 16.97$

Calculate A_{50}.

Question No. 7:

ABC Insurance Company sells 500 fully discrete whole life insurance policies of 1, each with the same age x. You are given:

- All policies have independent future lifetimes.
- $i = 0.05$
- $A_x = 0.270$
- ${}^2A_x = 0.093$
- Premium is determined according to the portfolio percentile principle, with the probability that the present value of the total future loss on the portfolio is negative is at least 95%.
- The 95th percentile of a standard normal distribution is 1.645.

Calculate the annual premium for each policy.

Question No. 8:

For a 20-year endowment life insurance policy issued to (45), you are given:

- The death benefit of 1000 is payable at the end of the year of death.
- A level premium is paid at the beginning of each year during the term of the policy.
- Mortality follows the `Survival Ultimate Life Table`.
- $i = 0.05$
- Net premium is calculated according to the actuarial equivalence principle.

Calculate the net premium reserve at the end of year 10.

Question No. 9:

For a fully discrete whole life insurance policy of 10,000 issued to (45), you are given:

- The only expense, incurred at policy issue, is 100.
- Mortality follows the `Survival Ultimate Life Table`.
- $i = 0.05$
- Gross premium is determined according to the actuarial equivalence principle.

Calculate the gross premium reserve at the end of year 10.

Question No. 10:

For a special single premium 20-year term insurance on (65):

- The death benefit, payable at the end of the year of death, is equal to 2500 plus the net premium reserve.
- $q_{65+k} = 0.02$, for $k = 0, 1, 2, \ldots$
- $i = 0.02$

Calculate the single net premium for this insurance.

Question No. 11:

For a fully discrete whole life insurance of 1000 to (50), you are given:

- Expenses consist of 10% of the annual gross premium in the first year and 5% of the annual gross premium in subsequent years.
- Mortality follows the `Survival Ultimate Life Table`.
- $i = 0.05$
- The annual gross premium is 17.50.

Calculate the probability of a positive loss at issue.

Question No. 12:

For a fully discrete whole life insurance of 10,000 on (45), you are given:

- The annual benefit premium is 161.45.
- The net premium reserve at the end of 15 years is ${}_{15}V = 607.55$.
- $q_{59} = 0.016$ and $q_{60} = 0.018$
- $i = 0.065$
- Deaths are uniformly distributed over integer ages.

Calculate ${}_{15.6}V$, the net premium reserve at the end of 15.6 years.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK