MATH 3630

Actuarial Mathematics I

Final Examination

Tuesday, 11 December 2018

Time Allowed: 2 hours (1:00 - 3:00 pm)

Room: OAK 117

Total Marks: 120 points

Please write your name and student number at the spaces provided:

Name: ______________________________ **Student ID:** ______________________

- **There are twelve (12) written-answer questions here and you are to answer all twelve. Each question is worth 10 points. Your final mark will be divided by 120 to convert to a unit of 100%.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught cheating will be subject to university's disciplinary action.**
- **Good luck.**
- **Have a Happy and Healthy Christmas and New Year!**

Question	Worth	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total	120	
%	$\div$ 120	

Question No. 1:

The mortality pattern for a cohort of newborn can be described by

$$\mu_x = \begin{cases} 0.01, & \text{for } 0 < x \le 40 \\ 0.04, & \text{for } x > 40 \end{cases} \longrightarrow \text{reduces to } .75(.04) = .03$$

A medical breakthrough reduces the force of mortality for age beyond 40 by 25%, but will not affect mortality prior to, and including, age 40.

Calculate the percentage improvement in the probability of a 25-year-old reaching to age 65 as a result of this medical breakthrough.

Without medical breakthrough

25 40 65

$${}_{40}p_{25} = {}_{15}p_{25}\ {}_{25}p_{40} = e^{-.01(15)}\, e^{-.04(25)} = 0.3166368$$

With medical breakthrough

$${}_{40}p_{25}^{MB} = e^{-.01(15)}\ {}_{25}p_{40}^{MB} = e^{-.01(15)}\, e^{-\cancel{.01}\,.03(25)} = \cancel{0.67032}\ 0.4065697$$

$$\frac{\cancel{.67032}\ .4065697}{.3166368} - 1 = \cancel{1.117}\ 0.284$$

~~112%~~ 28.4% improvement

Question No. 2:

For a life (65), you are given the following extract from a life table:

k	ℓ_{65+k}
0	5,000
1	4,900
2	4,700
3	4,400
4	4,000
5	3,500

Mortality between integer ages is assumed to follow Uniform Distribution of Death (UDD).

Calculate the probability that (66) will die between ages 68.25 and 69.40.

$$\begin{array}{r} 69.80 \\ 68.25 \\ \hline 1.65 \end{array}$$

66 68.25 69.80

2.25 1.65

$${}_{2.25|1.65}q_{66} = \frac{l_{68.25} - l_{69.80}}{l_{66}}$$

linearly interpolate

$$= \frac{(.25 l_{69} + .75 l_{68}) - (.80 l_{70} + .20 l_{69})}{l_{66}}$$

$$= \frac{700}{4900} = \frac{7}{49} = \frac{1}{7} = .142857$$

$l_{66} = 4900$

$l_{68} = 4400$

$l_{69} = 4000$

$l_{70} = 3500$

Question No. 3:

You are given:

- For age prior to 45, mortality follows a constant force with $\mu = 0.02$.
- For ages 45 and later, mortality follows the `Survival Ultimate Life Table`.
- $i = 0.05$
- Z is the present value random variable for a whole life insurance of 1 payable at the end of the year of death issued to (40).

Calculate the probability that Z will be greater than 0.6.

$\delta = \log(1.05)$

$$Z = v^{K+1} > .6 \Rightarrow (K+1)\underbrace{\log v}_{-\delta} > \log(.6)$$

$$K < \frac{\log(.6)}{-\delta} - 1 = 9.469848$$

$$\Pr(K < 9.469848) = \Pr(K \leq 9)$$

$$= {}_{10}q_{40}$$

$$= 1 - {}_{10}p_{40}$$

$$= 1 - {}_5p_{40}\, {}_5p_{45}$$

$$= 1 - e^{-.02(5)} \frac{l_{50}}{l_{45}} = \underline{0.0993426}$$

Question No. 4:

You are given:

- The following extract from a mortality table:

x	95	96	97	98	99	100
ℓ_x	1000	750	-	200	50	0

$490 = \ell_{97}$

given $\ddot{a}_{97} = 1.3$

- $v = 0.90$
- $\ddot{a}_{95} = 2.2$

Calculate $\ddot{a}_{97}$.

ask ℓ_{97}

$$\ddot{a}_{95} = 1 + v p_{95} + v^2 p_{95} p_{96} \ddot{a}_{97}$$

$$\ddot{a}_{97} = \frac{\ddot{a}_{95} - 1 - v p_{95}}{v^2 p_{95} p_{96}} = \frac{2.2 - 1 - .9\left(750/1000\right)}{(.9)^2\left(x/1000\right)} = 1.322751$$

490

round to 1.3

$$\frac{x}{1000} = \frac{\ddot{a}_{95} - 1 - v p_{95}}{(.9)^2 \ddot{a}_{97}} 1000$$

498.5755

Question No. 5:

A fully discrete whole life insurance of 1000 is issued to (46). You are given:

- Expenses consist of 10% of annual gross premium in the first year and 5% in subsequent years.
- $A_{45} = 0.15$
- $p_{45} = 0.99$
- $i = 0.04$

Calculate the gross annual premium for this policy.

$$G\ddot{a}_{46} = 1000A_{46} + .05G + .05G\ddot{a}_{46}$$

$$G(.95\ddot{a}_{46} - .05) = 1000A_{46}$$

$$A_{45} = vq_{45} + vp_{45}A_{46} \Rightarrow$$

$$A_{46} = \frac{\overset{0.15}{A_{45}} - \overset{\frac{1}{1.04}}{v}\,\overset{.01}{q_{45}}}{\underset{\frac{1}{1.04}}{v}\,\underset{.99}{p_{45}}} = \underline{.1474747}$$

$$\ddot{a}_{46} = \frac{1 - A_{46}}{d} = 22.16566$$

$$G = \frac{1000A_{46}}{.95\ddot{a}_{46} - .05} = \underline{7.020142}$$

Question No. 6:

For a special whole life insurance on (50), you are given:

- Death benefit, payable at the end of the year of death, consists of 1000 plus the return of all premiums without interest.
- Annual net premium of 16.95 is payable at the beginning of each year.
- $i = 0.05$
- $(IA)_{50} = 16.97$

Calculate A_{50}.

$$APV(FP) = APV(FB)$$

$$16.95\,\ddot{a}_{50} = 1000\,A_{50} + 16.95\,(IA)_{50}$$

$$\ddot{a}_{50} = \frac{1-A_{50}}{d}$$

$$\left(1000 + \frac{16.95}{d}\right)A_{50} = \frac{16.95}{d} - 16.95\,(IA)_{50}$$

$$(IA)_{50} = 16.97$$

$$A_{50} = \underbrace{\frac{\frac{16.95}{d}\left(1 - d\,(IA)_{50}\right)}{1000 + \frac{16.95}{d}}}_{.05037686}$$

Question No. 7:

$L_{0,i} = v^{K+1} - P\ddot{a}_{\overline{K+1}|} = v^{K+1}\left(1+\frac{P}{d}\right) - \frac{P}{d}$

ABC Insurance Company sells 500 fully discrete whole life insurance policies of 1, each with the same age x. You are given:

$d = .05/1.05 \quad A_x = .270 \quad \ddot{a}_x = 15.33$

- All policies have independent future lifetimes.
- $i = 0.05$
- $A_x = 0.270$
- ${}^2A_x = 0.093$
- Premium is determined according to the portfolio percentile principle, with the probability that the present value of the total future loss on the portfolio is negative is at least 95%.
- The 95th percentile of a standard normal distribution is 1.645.

$E[L_{0,i}] = A_x - P\ddot{a}_x = .270 - P(15.33)$

$Var[L_{0,i}] = \left(1+\frac{P}{d}\right)^2\left(.093 - (.27)^2\right)$

Calculate the annual premium for each policy.

$E(L_{agg}) = 500\left(.27 - P(15.33)\right)$

$Var(L_{agg}) = 500\left(1+\frac{P}{d}\right)^2\left(.093 - .27^2\right)$

$Pr[L_{agg} < 0] = .95$

$N < \frac{-E[L_{agg}]}{\sqrt{Var(L_{agg})}} = 1.645 \Rightarrow$ ~~$-E[L_{agg}] = 1.645$~~

$-500(.27 - 15.33P) = 1.645\sqrt{500}\left(1+\frac{P}{d}\right)\underbrace{\sqrt{.093 - .27^2}}_{\sqrt{.0201}}$

~~$500(15.33) - 1.645\frac{\sqrt{500}}{d}\sqrt{.0201} = 500(.27) + 1.645$~~

$\left(500(15.33) - \underbrace{1.645\frac{\sqrt{500}}{d}}_{temp}\sqrt{.0201}\right)P = 500(.27) + \underbrace{1.645\frac{\sqrt{500}}{d}}_{temp}\sqrt{.0201}$

$P = \frac{500(.27) + temp}{500(15.33) - temp} \approx .0323624$ more expensive than net premium

Question No. 8:

For a 20-year endowment life insurance policy issued to (45), you are given:

- The death benefit of 1000 is payable at the end of the year of death.
- A level premium is paid at the beginning of each year during the term of the policy.
- Mortality follows the `Survival Ultimate Life Table`.
- $i = 0.05$
- Net premium is calculated according to the actuarial equivalence principle.

Calculate the net premium reserve at the end of year 10.

$$P = 1000 A_{45:\overline{20}|} / \ddot{a}_{45:\overline{20}|} = \underline{29.6659}$$

$$A_{45:\overline{20}|} = .38385 \qquad \ddot{a}_{45:\overline{20}|} = 12.9391$$

$$_{10}V^n = 1000 A_{55:\overline{10}|} - P \ddot{a}_{55:\overline{10}|} = \underline{380.2332}$$

$$A_{55:\overline{10}|} = .61813 \qquad \ddot{a}_{55:\overline{10}|} = 8.0192$$

Question No. 9:

For a fully discrete whole life insurance policy of 10,000 issued to (45), you are given:

- The only expense, incurred at policy issue, is 100.
- Mortality follows the `Survival Ultimate Life Table`.
- $i = 0.05$
- Gross premium is determined according to the actuarial equivalence principle.

Calculate the gross premium reserve at the end of year 10.

$$G\ddot{a}_{45} = 10000 A_{45} + 100$$

$$G = 10000 \frac{A_{45}}{\ddot{a}_{45}} + \frac{100}{\ddot{a}_{45}} = P + \frac{100}{\ddot{a}_{45}}$$

$$P = 10000 A_{45}/\ddot{a}_{45} \quad 85.09671$$

$${}_{10}V^g = 10000 A_{55} - G\ddot{a}_{45} = \underbrace{10000 A_{55} - P\ddot{a}_{45} - 100}_{970.8521}$$

$A_{45} = .15161$

$A_{55} = .23524$

$\ddot{a}_{45} = 17.8162$

$\ddot{a}_{55} = 16.0599$

Question No. 10:

For a special single premium 20-year term insurance on (65):

- The death benefit, payable at the end of the year of death, is equal to 2500 plus the net premium reserve.
- $q_{65+k} = 0.02$, for $k = 0, 1, 2, \ldots$
- $i = 0.02$

Calculate the single net premium for this insurance.

20 year term $\quad {}_{20}V = 0$

${}_0V = 0$

$${}_1V = (0 + P)(1.02) - .02(2500 - \cancel{{}_1V} + \cancel{{}_1V})$$

$$= P(1.02) - 2500(.02)$$

$${}_2V = ({}_1V + P)(1.02) - .02(2500 - \cancel{{}_2V} + \cancel{{}_2V})$$

$$= P(1.02^2 + 1.02) - 2500(.02)(1 + 1.02)$$

$\vdots$

$${}_{20}V = P(1.02^{20} + \ldots + 1.02) - 2500(.02)(1 + 1.02 + \ldots + 1.02^{19}) = 0$$

$$P = \frac{2500(.02)\,\cancel{\frac{-1 + 1.02^{20}}{.02}}}{1.02\,\cancel{\frac{-1 + 1.02^{20}}{.02}}} = \frac{2500(.02)}{1.02} = \underline{\underline{49.01961}}$$

Question No. 11:

For a fully discrete whole life insurance of 1000 to (50), you are given:

- Expenses consist of 10% of the annual gross premium in the first year and 5% of the annual gross premium in subsequent years.
- Mortality follows the `Survival Ultimate Life Table`.
- $i = 0.05$
- The annual gross premium is 17.50.

Calculate the probability of a positive loss at issue.

$$.95G\left(\frac{1-v^{K+1}}{d}\right)$$

$$L_0^g = 1000\,v^{K+1} + .05G - .95G\,\ddot{a}_{\overline{K+1}|}$$

$$= \left(1000 + \frac{.95G}{d}\right)v^{K+1} + .05G - \frac{.95G}{d} > 0$$

$$v^{K+1} > \underbrace{\frac{.95G/d - .05G}{1000 + .95G/d}}_{a} \qquad G = 17.50$$

$$(K+1)\log v > \log a \qquad a = .0258/303$$

$$K < \underbrace{\frac{\log a}{\log v}}_{26.75746} - 1 \qquad \log v = -\log(1.05)$$

$$Pr[L_0 > 0] = Pr[K < 26.75746] = Pr[K \le 26]$$

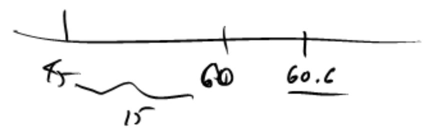
$$= {}_{27}q_{50} = 1 - \frac{\ell_{77}}{\ell_{50}} \qquad \ell_{77} = 81904.3 \quad \ell_{50} = 98576.4$$

$$= \underline{\underline{.169128\,7}}$$

Question No. 12:

For a fully discrete whole life insurance of 10,000 on (45), you are given:

- The annual benefit premium is 161.45.
- The net premium reserve at the end of 15 years is ${}_{15}V = 607.55$.
- $q_{59} = 0.016$ and $q_{60} = 0.018$
- $i = 0.065$
- Deaths are uniformly distributed over integer ages.



Calculate ${}_{15.6}V$, the net premium reserve at the end of 15.6 years.

$$q_{60} = .018$$
$${}_{15}V = 607.55$$
$$P = 161.45$$

$${}_{15.6}V = \frac{({}_{15}V + P)(1.065)^{.6} - 10000\,{}_{.6}q_{60}\,v}{1 - {}_{.6}q_{60}}$$

$$= \frac{(607.55 + 161.45)(1.065)^{.6} - 10000(.6)(.018)\,v^{.4}}{1 - .6(.018)}$$

$$= \frac{693.298}{.9892}$$

$$= 700.8684$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK