MATH 3630
Actuarial Mathematics I
Final Examination
Wednesday, 16 December 2015
Time Allowed: 2 hours (3:30 - 5:30 pm)
Room: LH 305
Total Marks: 120 points

Please write your name and student number at the spaces provided:

Name: ______________________________ **Student ID:** ______________________

- **There are twelve (12) written-answer questions here and you are to answer all twelve. Each question is worth 10 points. Your final mark will be divided by 120 to convert to a unit of 100%.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**
- **Best of luck.**
- **Have a Happy and Healthy Christmas and New Year!**

Question	Worth	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total	120	
%	÷ 120	

Question No. 1:

Suppose that mortality follows the Makeham's law:

$$\mu_x = A + Bc^x, \quad \text{for } x \geq 0.$$

You are given

- ${}_5p_{40} = 0.9392$
- ${}_5p_{45} = 0.8954$
- ${}_5p_{50} = 0.8232$

Calculate c.

Question No. 2:

In a two-year select and ultimate mortality table, you are given:

- For some positive constant b, we have

$$q_{[x]} = (1-2b) \times q_x \quad \text{and} \quad q_{[x]+1} = (1-b) \times q_{x+1}$$

  for all $x \geq 0$.

- $\ell_{[40]} = 9,000$
- $\ell_{40} = 10,000 \qquad \ell_{41} = 9,000 \qquad \ell_{42} = 6,300$

Calculate $\ell_{[40]+1}$.

Question No. 3:

You are given:

- For age prior to 50, mortality follows a constant force with $\mu = 0.005$.
- For ages 65 and later, mortality is uniformly distributed with $\omega = 110$.
- $\delta = 5\%$
- Z is the present value random variable for a whole life insurance of \$1 issued to (40), with benefit payable at the end of the year of death.

Calculate the probability that Z will be greater than 0.45.

Question No. 4:

For a whole life annuity-due issued to (45), you are given:

- For age prior to 65, deaths are uniformly distributed with ${}_{20}p_{45} = 0.5$.
- $\delta = 0.05$
- $A_{65} = 0.40$

Calculate $\ddot{a}_{65}$.

Question No. 5:

For a fully discrete whole life insurance of \$1 issued to (x), you are given:

- L_0 is the loss at issue random variable with the premium determined according to the actuarial equivalence principle.
- L_0^* is the loss at issue random variable with the premium determined such that $\mathrm{E}[L_0^*] = c$, for some constant c.
- $\mathrm{Var}[L_0] = 0.36$
- $\mathrm{Var}[L_0^*] = 0.45$

Calculate c.

Question No. 6:

For a special fully discrete 2-year term insurance policy issued to (63), you are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 3\%$
- The death benefit is \$500 plus a return of all premiums paid without interest.
- Premiums are calculated based on the actuarial equivalence principle.

Calculate the net annual premium for this policy.

Question No. 7:

Get-a-Life Insurance Company issues a special insurance policy to (50) with the following benefits:

- A death benefit of 100, payable at the end of year of death, provided death occurs before age 65.
- An annuity benefit that pays 500 annually starting immediately when the policyholder reaches age 65.

You are given:

- Level annual premiums of P are paid for the first 15 years only and are determined according to the actuarial equivalence principle.
- $i = 0.05$
- $\ddot{a}_{50} = 19.00$
- $\ddot{a}_{65} = 18.35$
- ${}_{15}E_{50} = 0.45$

Calculate P.

Question No. 8:

For a fully discrete whole life insurance policy of \$1 on (40), you are given:

- Net annual premium is calculated according to the equivalence principle.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$

Calculate the probability that the policy makes a positive loss.

Question No. 9:

A 30,000 fully discrete whole life policy issued to (35) with level annual premiums is priced with the following expense assumptions:

	% of Premium	Per 1,000	Per Policy
First year	25%	1.0	50
Renewal years	10%	0.3	25

You are given:

- $i = 0.03$
- $\ddot{a}_{35} = 28.98$

Calculate the gross annual premium.

Question No. 10:

For a fully discrete whole life insurance of \$1,000 issued to (x), you are given:

- The net annual premium is \$12.
- Based on the normal approximation, if n of such policies with independent future lifetimes, were sold, the probability of a loss will be 0.02.
- $i = 0.05$
- $A_x = 0.24$
- ${}^2A_x = 0.64$
- The 98th percentile of the standard normal distribution is 2.054.

Calculate n.

Question No. 11:

For a fully discrete 10-year endowment life insurance of \$100 issued to (35), you are given:

- Level gross annual premiums are calculated according to the equivalence principle.
- The first year expense is 20% of the gross annual premium.
- Expenses in subsequent years are 5% of the gross annual premium.
- $i = 0.05$
- $A_{35:\overline{10|}} = 0.60$
- ${}^2A_{35:\overline{10|}} = 0.40$
- L_0^g is the gross loss at issue random variable.

Calculate $\text{Var}[L_0^g]$.

Question No. 12:

Based on the same mortality and interest assumptions, you are given:

- $\ddot{a}_{65}^{(4)} = 10.70191$ using the Woolhouse's approximation with two terms.
- $\ddot{a}_{65}^{(12)} = 10.61361$ using the Woolhouse's approximation with three terms.

Calculate $\ddot{a}_{65}^{(6)}$ using the Woolhouse's approximation with three terms.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK