MATH 3630
Actuarial Mathematics I
Final Examination
Wednesday, 16 December 2015
Time Allowed: 2 hours (3:30 - 5:30 pm)
Room: LH 305
Total Marks: 120 points

Please write your name and student number at the spaces provided:

Name: EMIL

Student ID: Suggested Solutions

- There are twelve (12) written-answer questions here and you are to answer all twelve. Each question is worth 10 points. Your final mark will be divided by 120 to convert to a unit of 100%.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.
- Best of luck.
- Have a Happy and Healthy Christmas and New Year!

Question	Worth	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total	120	
%	÷ 120	

Question No. 1:

Suppose that mortality follows the Makeham's law:

$$\mu_x = A + Bc^x, \quad \text{for } x \geq 0.$$

You are given:

- ${}_5p_{40} = 0.9392$
- ${}_5p_{45} = 0.8954$
- ${}_5p_{50} = 0.8232$

Calculate c.

It can be shown that

$$ {}_tp_x = \exp\left\{-\int_0^t \mu_{x+s}\,ds\right\} = \exp\left\{-At - \frac{Bc^x}{\log c}(c^t-1)\right\}$$

So that $\log {}_tp_x = -At - \dfrac{Bc^x}{\log c}(c^t-1)$

Thus, $$\log {}_5p_{40} - \log {}_5p_{45} = \frac{Bc^{40}}{\log c}(c^5-1)(c^5-1)$$

$$\log {}_5p_{45} - \log {}_5p_{50} = \frac{Bc^{45}}{\log c}(c^5-1)(c^5-1)$$

Divide both equations, we get

$$\frac{\log(.9392/.8954)}{\log(.8954/.8232)} = \frac{1}{c^5}$$

Solving for c, we get

$$c = \left[\frac{\log(.9392/.8954)}{\log(.8954/.8232)}\right]^{-1/5} = 1.119749 \approx \underline{\underline{1.12}}$$

Question No. 2:

In a two-year select and ultimate mortality table, you are given:

- For some positive constant b, we have
$$q_{[x]} = (1-2b) \times q_x \quad \text{and} \quad q_{[x]+1} = (1-b) \times q_{x+1}$$
for all $x \geq 0$.
- $\ell_{[40]} = 9{,}000$
- $\ell_{40} = 10{,}000 \qquad \ell_{41} = 9{,}000 \qquad \ell_{42} = 6{,}300$

Calculate $\ell_{[40]+1}$.

$q_{40} = 1 - \frac{\ell_{41}}{\ell_{40}} = 1 - \frac{9000}{10000} = \frac{1}{10}$

Consider $p_{[40]} = \frac{\ell_{[40]+1}}{\ell_{[40]}} = 1-(1-2b)q_{40} \Rightarrow \ell_{[40]+1} = 9000\left[1-(1-2b)\frac{1}{10}\right] = 8100 - 1800b$

$q_{41} = 1 - \frac{\ell_{42}}{\ell_{41}} = \frac{3}{10}$

and $p_{[40]+1} = \frac{\overset{6300}{\ell_{[40]+2}}}{\ell_{[40]+1}} = 1-(1-b)q_{41} \Rightarrow 6300 = \ell_{[40]+1}\left[1-(1-b)\frac{3}{10}\right]$

Thus, we have

$$630\cancel{0} = (810\cancel{0} - 180\cancel{0}b)\left[1-(1-b)\frac{3}{10}\right]$$

$$\Rightarrow 630 = 567 + 243b + 126b + 54b^2$$

$$\Rightarrow 54b^2 + 369b - 63 = 0 \Rightarrow b = \frac{-369 \pm \sqrt{369^2 - 4(54)(-63)}}{2(54)}$$

Choose $b = \frac{18}{108} = \frac{1}{6}$

Finally,

$$\ell_{[40]+1} = 9000\left[\underbrace{1-\left(1-\frac{1}{3}\right)\frac{1}{10}}_{\frac{28}{30}}\right] = \underline{\underline{8400}}$$

Question No. 3:

You are given:

- For age prior to 50, mortality follows a constant force with $\mu = 0.005$.
- For ages 50 and later, mortality is uniformly distributed with $\omega = 110$.
- $\delta = 5\%$
- Z is the present value random variable for a whole life insurance of \$1 issued to (40), with benefit payable at the end of the year of death.

Calculate the probability that Z will be greater than 0.45.

Let $T = T_{40}$ be the future lifetime of (40). Its density can be expressed as

$$f_T(t) = \begin{cases} .005 e^{-.005t}, & 0 \le t < 10 \\ e^{-.05} \frac{1}{60}, & 10 \le t < 70 \end{cases}$$

Thus, the event $Z = v^{K+1} > 0.45$ is equivalent to

$$(K+1) \underbrace{\log v}_{-\delta} > \log 0.45 \Rightarrow K < \underbrace{\frac{\log(0.45)}{-\delta} - 1}_{\text{with } \delta = .05}$$

14.97015

$$\Pr[Z > 0.45] = \Pr[K < 14.97015] = \Pr[K \le 14]$$

$$= \Pr[T < 15] = \int_0^{10} .005 e^{-.005t} dt + \int_{10}^{15} e^{-.05} \frac{1}{60} dt$$

$$= 1 - e^{-.05} + \frac{5}{60} e^{-.05}$$

$$= 1 - \frac{55}{60} e^{-.05} = 1 - \frac{11}{12} e^{-.05} = 0.1280397$$

Question No. 4:

For a whole life annuity-due issued to (45), you are given:

- For age prior to 65, deaths are uniformly distributed with ${}_{20}p_{45} = 0.5$.
- $\delta = 0.05$
- $A_{65} = 0.40$

Calculate $\ddot{a}_{45}$.

The density of the future lifetime of (45) can be expressed as

$$f_T(t) = \begin{cases} \frac{1}{2}\frac{1}{20} = \frac{1}{40}, & 0 \le t < 20 \\ \text{unknown}, & t \geq 20 \end{cases}$$

$$A^{1}_{45:\overline{20|}} = \sum_{K=0}^{19} v^{K+1} \underbrace{{}_{K}p_x\, q_{x+K}}_{\frac{1}{40}} = \frac{1}{40} \cdot \cancel{v}\frac{(1-v^{20})}{\cancel{1-v}\; i}$$

$$= 0.3082246$$

$$A_{45} = A^{1}_{45:\overline{20|}} + \underbrace{{}_{20}E_{45}}_{v^{20}\,0.5}\; \underbrace{A_{65}}_{0.40} \qquad {}_{20}p_{45} = 0.5$$

$$= 0.3082246 + \exp(20(.05))(0.5)(0.40) = 0.3818005$$

$$\ddot{a}_{45} = \frac{1-A_{45}}{d} = \frac{1-0.3818005}{1-\exp(-.05)} = \underline{\underline{12.67567}}$$

Question No. 5:

For a fully discrete whole life insurance of $1 issued to (x), you are given:

- L_0 is the loss at issue random variable with the premium determined according to the actuarial equivalence principle.
- L_0^* is the loss at issue random variable with the premium determined such that $\mathrm{E}\big[L_0^*\big] = c$, for some constant c.
- $\mathrm{Var}\big[L_0\big] = 0.36$.
- $\mathrm{Var}\big[L_0^*\big] = 0.45$.

Calculate c.

$$L_0 = v^{K+1} - P\ddot{a}_{\overline{K+1}|} = \left(1+\frac{P}{d}\right)v^{K+1} - \frac{P}{d}$$

$$\frac{\mathrm{Var}[L_0]}{\mathrm{Var}[L_0^*]} = \frac{\left(1+\frac{P}{d}\right)^2 \cancel{\mathrm{Var}[v^{K+1}]}}{\left(1+\frac{P^*}{d}\right)^2 \cancel{\mathrm{Var}[v^{K+1}]}}$$

where $P = A_x/\ddot{a}_x$ and P^* is the solution to

$$A_x - P^*\ddot{a}_x = c$$

$$\Rightarrow P^* = \frac{A_x - c}{\ddot{a}_x}$$

$$= \left(\frac{1+\frac{A_x}{d\ddot{a}_x}}{1+\frac{A_x-c}{d\ddot{a}_x}}\right)^2 = \left(\frac{d\ddot{a}_x + A_x}{d\ddot{a}_x + A_x - c}\right)^2 = \left(\frac{1}{1-c}\right)^2 = \frac{.36}{.45} = \frac{4}{5}$$

$$\Rightarrow 1-c = \pm\sqrt{\frac{5}{4}} \Rightarrow c = 1 \pm \sqrt{\frac{5}{4}}$$

but c cannot be greater than 1

Choose $c = 1 - \sqrt{\frac{5}{4}} = \underline{\underline{-0.118034}}$

loss on the average for P^*!

Question No. 6:

For a special fully discrete 2-year term insurance policy issued to (63), you are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 3\%$
- The death benefit is \$500 plus a return of all premiums paid without interest.
- Premiums are calculated based on the actuarial equivalence principle.

Calculate the net annual premium for this policy.

Let P = net annual premium

63 64 65

$$\text{APV(premiums)} = P(1 + v\,p_{63})$$

$$\text{APV(benefits)} = (500 + P)\,v\,q_{63} + (500 + 2P)\,v^2\,p_{63}\,q_{64}$$

$$= 500(v\,q_{63} + v^2\,p_{63}\,q_{64}) + P(v\,q_{63} + 2v^2\,p_{63}\,q_{64})$$

Equating and solving for P, we get

$$P = \frac{500\frac{1}{1.03}\left(\frac{17.88}{1000} + \frac{1}{1.03}\left(1 - \frac{17.88}{1000}\right)\left(\frac{19.52}{1000}\right)\right)}{1 + \frac{1}{1.03}\left(1 - \frac{17.88}{1000} - \frac{17.88}{1000}\right) - 2v^2\left(1 - \frac{17.88}{1000}\right)\left(\frac{19.52}{1000}\right)}$$

$$= \frac{17.71486}{1.900014} \approx \underline{\underline{9.323538}}$$

Question No. 7:

Get-a-Life Insurance Company issues a special insurance policy to (50) with the following benefits:

- A death benefit of 100, payable at the end of year of death, provided death occurs before age 65.
- An annuity benefit that pays 500 annually starting immediately when the policyholder reaches age 65.

You are given:

- Level annual premiums of P are paid for the first 15 years only and are determined according to the actuarial equivalence principle.
- $i = 0.05$
- $\ddot{a}_{50} = 19.00$
- $\ddot{a}_{65} = 18.35$
- ${}_{15}E_{50} = 0.45$

Calculate P.

$$APV(\text{benefits}) = 100\, A^{1}_{50:\overline{15|}} + {}_{15}E_{50}\, \ddot{a}_{65} \cdot 500$$

$$APV(\text{premium}) = P\, \ddot{a}_{50:\overline{15|}}$$

$$A_{50} = 1 - d\ddot{a}_{50} = .0952381 \qquad A_{65} = 1 - d\ddot{a}_{65} = 0.1261905$$

Solving for P, we get

$$P = \frac{100\left(.0952381 - (.45)(.1261905)\right) + 500(.45)(18.35)}{19.00 - 0.45(18.35)}$$

$$= 4132.595 / 10.7425$$

$$= \underline{\underline{384.6959}}$$

Question No. 8:

For a fully discrete whole life insurance policy of $1 on (40), you are given:

- Net annual premium is calculated according to the equivalence principle.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$

Calculate the probability that the policy makes a positive loss.

The net annual premium is $P = \frac{A_{40}}{\ddot{a}_{40}} = \frac{0.16132}{14.8166} = 0.01088779$

$$L_0 = v^{K+1} - P\ddot{a}_{\overline{K+1}|} = \left(1+\frac{P}{d}\right)v^{K+1} - P/d$$

$$\Pr[L_0 > 0] = \Pr\left[v^{K+1} > \frac{P/d}{1+P/d}\right]$$

$$= \Pr\left[(K+1)(\cancel{-\delta}) < \frac{\log\left((P/d)/(1+P/d)\right)}{-\delta}\right]$$

$$= \Pr\left[K < \underbrace{\frac{\log\left(\frac{P/d}{1+P/d}\right)}{-\delta} - 1}_{30.30934}\right]$$

$$= \Pr[K \leq 30]$$

$$= {}_{31}q_{40} = 1 - \frac{\ell_{71}}{\ell_{40}} = 1 - \frac{6396609}{9313166}$$

$$= \underline{\underline{0.3131649}}$$

Question No. 9:

A 30,000 fully discrete whole life policy issued to (35) with level annual premiums is priced with the following expense assumptions:

	% of Premium	Per 1,000	Per Policy
First year	25%	1.0	50
Renewal years	10%	0.3	25

You are given:

- $i = 0.03$
- $\ddot{a}_{35} = 28.98$

Calculate the gross annual premium.

$$A_{35} = 1 - d\,\ddot{a}_{35} = 1 - \frac{.03}{1.03}\,28.98 = 0.1559223$$

$$APV(\text{premiums}) = APV(\text{benefits}) + APV(\text{expenses})$$

$$G\ddot{a}_{35} = 30000A_{35} + 0.15G + 0.10G\ddot{a}_{35}$$
$$+ \overset{\times 30}{0.70} + \overset{\times 30}{0.30}\,\ddot{a}_{35}$$
$$+ 25 + 25\ddot{a}_{35}$$

$$\cancel{G(0.90\,\ddot{a}_{35} - 0.15)} = \frac{30000A_{35} + 46 + 34\ddot{a}_{35}}{0.90\,\ddot{a}_{35} - 0.15}$$

$$G = \frac{30000(0.1559223) + 46 + 34(28.98)}{0.90(28.98) - 0.15}$$

$$= \frac{5708.99}{25.932} = \underline{\underline{220.1523}}$$

Question No. 10:

For a fully discrete whole life insurance of \$1,000 issued to (x), you are given:

- The net annual premium is \$12.
- Based on the normal approximation, if n of such policies with independent future lifetimes, were sold, the probability of a loss will be 0.02.
- $i = 0.05$
- $A_x = 0.24$
- ${}^2A_x = 0.64$
- The 98th percentile of the standard normal distribution is 2.054.

Calculate n.

For one such policy, $L_0 = 1000v^{K+1} - 12\ddot{a}_{\overline{K+1|}} = \left(1000 + \frac{12}{d}\right)v^{K+1} - \frac{12}{d}$

$$E[L_0] = \left(1000 + \frac{12}{d}\right)A_x - \frac{12}{d} = 48.48$$

$$Var[L_0] = \left(1000 + \frac{12}{d}\right)^2\left({}^2A_x - A_x^2\right) = 912914.3$$

$$\Pr\left[\frac{L_0 - E[L_0]n}{\sqrt{Var[L_0]n}} > \underbrace{\frac{0 - 48.48n}{\sqrt{912914.3n}}}_{2.054}\right] = 0.02$$

$$\Rightarrow \quad -48.48\sqrt{n} = 2.054\sqrt{912914.3}$$

$$\Rightarrow \quad (48.48)^2 n = \left(2.054\sqrt{912914.3}\right)^2$$

$$\Rightarrow \quad n = \left(\frac{1962.526}{48.48}\right)^2 = 1638.723$$

Choose $n = 1639$

Question No. 11:

For a fully discrete 10-year endowment life insurance of \$100 issued to (35), you are given:

- Level gross annual premiums are calculated according to the equivalence principle.
- The first year expense is 20% of the gross annual premium.
- Expenses in subsequent years are 5% of the gross annual premium.
- $i = 0.05$
- $A_{35:\overline{10}|} = 0.60$
- ${}^2A_{35:\overline{10}|} = 0.40$
- L_0^g is the gross loss at issue random variable.

Calculate $\mathrm{Var}[L_0^g]$.

$$L_0^g = 100\, v^{\min(K+1,10)} - G\, \ddot{a}_{\overline{\min(K+1,10)}|} + 0.15G + .05G\, \ddot{a}_{\overline{\min(K+1,n)}|}$$

$$= \left(100 + \frac{.95G}{d}\right) v^{\min(K+1,n)} + \text{constant}$$

$$\mathrm{Var}[L_0^g] = \left(100 + \frac{.95G}{d}\right)^2 \left({}^2A_{35:\overline{10}|} - \left(A_{35:\overline{10}|}\right)^2\right)$$

$$G = \frac{100\, \overbrace{A_{35:\overline{10}|}}^{.60}}{.85\, \underbrace{\ddot{a}_{35:\overline{10}|}}_{\frac{1-.60}{.05/1.05}} - .05} = 8.462623$$

$$= \left(100 + \frac{.95(8.462623)}{.05/1.05}\right)^2 (.40 - .60^2) = \underline{\underline{2890.768}}$$

Question No. 12:

Based on the same mortality and interest assumptions, you are given:

- $\ddot{a}_{65}^{(4)} = 10.70191$ using the Woolhouse's approximation with two terms.
- $\ddot{a}_{65}^{(12)} = 10.61361$ using the Woolhouse's approximation with three terms.

Calculate $\ddot{a}_{65}^{(6)}$ using the Woolhouse's approximation with three terms.

Recall: $\ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m} - \frac{m^2-1}{12m^2}(\mu_x + \delta)$

$$\ddot{a}_{65}^{(4)} = \ddot{a}_{65} - \frac{3}{8}$$

$$-) \quad \ddot{a}_{65}^{(12)} = \ddot{a}_{65} - \frac{11}{24} - \frac{143}{1728}(\mu_{65} + \delta)$$

$$\underbrace{10.70191 - 10.61361}_{.0883} = \left(\frac{11}{24} - \frac{3}{8}\right) + \frac{143}{1728}(\mu_{65} + \delta)$$

$$\Rightarrow \mu_{65} + \delta = \frac{.0883 - \left(\frac{11}{24} - \frac{3}{8}\right)}{143/1728} = .06001678$$

$$\ddot{a}_{65} = 10.70191 + \frac{3}{8} = 11.07691$$

$$\ddot{a}_{65}^{(6)} \approx \underset{11.07691}{\ddot{a}_{65}} - \frac{5}{12} - \frac{35}{432}(.06001678)$$

$$= \underline{\underline{10.65538}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$	$1000\,{}_{20}E_x$	x
0	10,000,000	20.42	16.8010	49.00	25.92	728.54	541.95	299.89	0
5	9,749,503	0.98	17.0379	35.59	8.45	743.89	553.48	305.90	5
10	9,705,588	0.85	16.9119	42.72	9.37	744.04	553.34	305.24	10
15	9,663,731	0.91	16.7384	52.55	11.33	743.71	552.69	303.96	15
20	9,617,802	1.03	16.5133	65.28	14.30	743.16	551.64	301.93	20
21	9,607,896	1.06	16.4611	68.24	15.06	743.01	551.36	301.40	21
22	9,597,695	1.10	16.4061	71.35	15.87	742.86	551.06	300.82	22
23	9,587,169	1.13	16.3484	74.62	16.76	742.68	550.73	300.19	23
24	9,576,288	1.18	16.2878	78.05	17.71	742.49	550.36	299.49	24
25	9,565,017	1.22	16.2242	81.65	18.75	742.29	549.97	298.73	25
26	9,553,319	1.27	16.1574	85.43	19.87	742.06	549.53	297.90	26
27	9,541,153	1.33	16.0873	89.40	21.07	741.81	549.05	297.00	27
28	9,528,475	1.39	16.0139	93.56	22.38	741.54	548.53	296.01	28
29	9,515,235	1.46	15.9368	97.92	23.79	741.24	547.96	294.92	29
30	9,501,381	1.53	15.8561	102.48	25.31	740.91	547.33	293.74	30
31	9,486,854	1.61	15.7716	107.27	26.95	740.55	546.65	292.45	31
32	9,471,591	1.70	15.6831	112.28	28.72	740.16	545.90	291.04	32
33	9,455,522	1.79	15.5906	117.51	30.63	739.72	545.07	289.50	33
34	9,438,571	1.90	15.4938	122.99	32.68	739.25	544.17	287.82	34
35	9,420,657	2.01	15.3926	128.72	34.88	738.73	543.18	286.00	35
36	9,401,688	2.14	15.2870	134.70	37.26	738.16	542.11	284.00	36
37	9,381,566	2.28	15.1767	140.94	39.81	737.54	540.92	281.84	37
38	9,360,184	2.43	15.0616	147.46	42.55	736.86	539.63	279.48	38
39	9,337,427	2.60	14.9416	154.25	45.48	736.11	538.22	276.92	39
40	9,313,166	2.78	14.8166	161.32	48.63	735.29	536.67	274.14	40
41	9,287,264	2.98	14.6864	168.69	52.01	734.40	534.99	271.12	41
42	9,259,571	3.20	14.5510	176.36	55.62	733.42	533.14	267.85	42
43	9,229,925	3.44	14.4102	184.33	59.48	732.34	531.12	264.31	43
44	9,198,149	3.71	14.2639	192.61	63.61	731.17	528.92	260.48	44
45	9,164,051	4.00	14.1121	201.20	68.02	729.88	526.52	256.34	45
46	9,127,426	4.31	13.9546	210.12	72.72	728.47	523.89	251.88	46
47	9,088,049	4.66	13.7914	219.36	77.73	726.93	521.03	247.08	47
48	9,045,679	5.04	13.6224	228.92	83.06	725.24	517.91	241.93	48
49	9,000,057	5.46	13.4475	238.82	88.73	723.39	514.51	236.39	49
50	8,950,901	5.92	13.2668	249.05	94.76	721.37	510.81	230.47	50
51	8,897,913	6.42	13.0803	259.61	101.15	719.17	506.78	224.15	51
52	8,840,770	6.97	12.8879	270.50	107.92	716.76	502.40	217.42	52
53	8,779,128	7.58	12.6896	281.72	115.09	714.12	497.64	210.27	53
54	8,712,621	8.24	12.4856	293.27	122.67	711.24	492.47	202.70	54
55	8,640,861	8.96	12.2758	305.14	130.67	708.10	486.86	194.72	55
56	8,563,435	9.75	12.0604	317.33	139.11	704.67	480.79	186.32	56
57	8,479,908	10.62	11.8395	329.84	147.99	700.93	474.22	177.53	57
58	8,389,826	11.58	11.6133	342.65	157.33	696.85	467.12	168.37	58
59	8,292,713	12.62	11.3818	355.75	167.13	692.41	459.46	158.87	59
60	8,188,074	13.76	11.1454	369.13	177.41	687.56	451.20	149.06	60
61	8,075,403	15.01	10.9041	382.79	188.17	682.29	442.31	139.00	61
62	7,954,179	16.38	10.6584	396.70	199.41	676.56	432.77	128.75	62
63	7,823,879	17.88	10.4084	410.85	211.13	670.33	422.54	118.38	63
64	7,683,979	19.52	10.1544	425.22	223.34	663.56	411.61	107.97	64
65	7,533,964	21.32	9.8969	439.80	236.03	656.23	399.94	97.60	65

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$	$1000\,{}_{20}E_x$	x
66	7,373,338	23.29	9.6362	454.56	249.20	648.27	387.53	87.37	66
67	7,201,635	25.44	9.3726	469.47	262.83	639.66	374.36	77.38	67
68	7,018,432	27.79	9.1066	484.53	276.92	630.35	360.44	67.74	68
69	6,823,367	30.37	8.8387	499.70	291.46	620.30	345.77	58.54	69
70	6,616,155	33.18	8.5693	514.95	306.42	609.46	330.37	49.88	70
71	6,396,609	36.26	8.2988	530.26	321.78	597.79	314.27	41.86	71
72	6,164,663	39.62	8.0278	545.60	337.54	585.25	297.51	34.53	72
73	5,920,394	43.30	7.7568	560.93	353.64	571.81	280.17	27.96	73
74	5,664,051	47.31	7.4864	576.24	370.08	557.43	262.31	22.19	74
75	5,396,081	51.69	7.2170	591.49	386.81	542.07	244.03	17.22	75
76	5,117,152	56.47	6.9493	606.65	403.80	525.71	225.46	13.04	76
77	4,828,182	61.68	6.6836	621.68	421.02	508.35	206.71	9.61	77
78	4,530,360	67.37	6.4207	636.56	438.42	489.97	187.94	6.88	78
79	4,225,163	73.56	6.1610	651.26	455.95	470.57	169.31	4.77	79
80	3,914,365	80.30	5.9050	665.75	473.59	450.19	151.00	3.19	80
81	3,600,038	87.64	5.6533	680.00	491.27	428.86	133.19	2.05	81
82	3,284,542	95.61	5.4063	693.98	508.96	406.62	116.06	1.27	82
83	2,970,496	104.28	5.1645	707.67	526.60	383.57	99.81	0.75	83
84	2,660,734	113.69	4.9282	721.04	544.15	359.79	84.59	0.42	84
85	2,358,246	123.89	4.6980	734.07	561.57	335.40	70.56	0.22	85
86	2,066,090	134.94	4.4742	746.74	578.80	310.56	57.83	0.11	86
87	1,787,299	146.89	4.2571	759.03	595.79	285.44	46.50	0.05	87
88	1,524,758	159.81	4.0470	770.92	612.51	260.21	36.61	0.02	88
89	1,281,083	173.75	3.8442	782.41	628.92	235.11	28.17	0.01	89
90	1,058,491	188.77	3.6488	793.46	644.96	210.36	21.13	0.00	90
91	858,676	204.93	3.4611	804.09	660.61	186.21	15.41	0.00	91
92	682,707	222.27	3.2812	814.27	675.83	162.90	10.91	0.00	92
93	530,959	240.86	3.1091	824.01	690.59	140.69	7.47	0.00	93
94	403,072	260.73	2.9450	833.30	704.86	119.79	4.93	0.00	94
95	297,981	281.91	2.7888	842.14	718.61	100.43	3.13	0.00	95
96	213,977	304.45	2.6406	850.53	731.83	82.78	1.90	0.00	96
97	148,832	328.34	2.5002	858.48	744.50	66.97	1.10	0.00	97
98	99,965	353.60	2.3676	865.99	756.60	53.09	0.60	0.00	98
99	64,617	380.20	2.2426	873.06	768.13	41.14	0.31	0.00	99
100	40,049	408.12	2.1252	879.70	779.08	31.12	0.15	0.00	100
101	23,705	437.28	2.0152	885.93	789.44	22.91	0.07	0.00	101
102	13,339	467.61	1.9123	891.76	799.21	16.37	0.03	0.00	102
103	7,101	498.99	1.8164	897.19	808.41	11.33	0.01	0.00	103
104	3,558	531.28	1.7273	902.23	817.02	7.56	0.00	0.00	104
105	1,668	564.29	1.6447	906.90	825.06	4.86	0.00	0.00	105
106	727	597.83	1.5685	911.22	832.53	2.99	0.00	0.00	106
107	292	631.64	1.4984	915.19	839.46	1.76	0.00	0.00	107
108	108	665.45	1.4341	918.82	845.84	0.98	0.00	0.00	108
109	36	698.97	1.3755	922.14	851.69	0.52	0.00	0.00	109
110	11	731.87	1.3223	925.15	857.04	0.26	0.00	0.00	110