MATH 3630
Actuarial Mathematics I
Class Test 1 - 3:35-4:50 PM
Wednesday, 15 November 2017
Time Allowed: 1 hour and 15 minutes
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: ______________________________ **Student ID:** ______________________

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught cheating will be subject to university's disciplinary action.**

Question No. 1:

For a special whole life insurance of 1 issued to (30) with benefits payable at the end of the year of death, you are given:

- Mortality follows the `Illustrative Life Table` except for:
  - ages between 35 and 45 where mortality has a constant force of 0.001.
- $i = 0.06$
- Z is the present value random variable for this insurance.

Calculate $\mathrm{Var}[Z]$.

Question No. 2:

You are given:

- Mortality follows a constant force of $\mu = 0.02$.
- $i = 0.05$
- Y is the present value random variable for a 3-year temporary life annuity-immediate of 1 per year on (x).

Calculate $\mathrm{Var}[Y]$.

Question No. 3:

For a group of 500 lives, each age 65, with independent future lifetimes, you are given:

- Each life is to be paid 5 per month at the beginning of each month, if alive.
- To fund these payments, each life will contribute an amount of c to a fund to support these payments. This contribution is to be made immediately today and only once.
- Y is the present value random variable today of total annuity payments to the 500 lives.
- $i^{(12)} = 0.12$
- $A_{65}^{(12)} = 0.1196$
- ${}^2A_{65}^{(12)} = 0.0395$
- The 95$^{\text{th}}$ percentile of a standard normal distribution is 1.645.

Using the normal approximation, calculate c such that $\Pr[500c > Y] = 0.95$.

Question No. 4:

Based on the same mortality and interest assumptions, you are given:

- $i = 0.06$
- $\ddot{a}_{35}^{(4)} = 13.9178$ using the Woolhouse's approximation with three terms.
- $\ddot{a}_{35}^{(6)} = 13.8759$ using the Woolhouse's approximation with three terms.

Calculate μ_{35}.

Question No. 5:

For a whole life annuity-due of 1 payable at the beginning of each year on (45), you are given:

- Mortality follows de Moivre's law with $\omega = 110$.
- $i = 0.10$
- Y is the present value random variable for this annuity.

Calculate the probability that Y exceeds 7.

Question No. 6:

For the country of Zooto, you are given:

- Zooto publishes mortality rates in 2-year intervals, that is mortality rates are of the form: ${}_2q_{2x}$, for $x = 0, 1, 2, \ldots$
- Deaths are assumed to be uniformly distributed between ages $2x$ and $2x + 2$, for $x = 0, 1, 2, \ldots$
- ${}_2p_{62} = 0.90$
- ${}_2p_{64} = 0.88$
- ${}_{3.75}p_{62.75} = 0.79097$

Calculate the probability that a person in Zooto now age 66 will die before reaching age 68.

Question No. 7:

You are given:

- The following select-and-ultimate mortality table with a 3-year select period:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	x+3
54	977	972	965	958	57
55	970	965	958	951	58
56	963	957	950	942	59

- Deaths are uniformly distributed between integral ages.
- $i = 0.04$
- $1000A_{[55]+2.5} = 535$

Calculate $1000\,{}_{2.5|}A_{[55]}$.

Question No. 8:

Tammy is age 65 and just newly retired. She has a total personal savings of F.

She wants guaranteed income while alive. In exchange for a single payment of F, an insurance company promised her an annual payment (at the beginning of each year) of 50,000 with:

- the first 10 payments guaranteed, whether she is alive or not, and
- the subsequent payments made provided she is alive.

You are given:

- $i = 0.05$
- $\ddot{a}_{65} = 10.263$
- $\ddot{a}_{75} = 7.448$
- $\ddot{a}_{65:\overline{10}|} = 7.095$

Calculate F.

Question No. 9:

You are given:

- Z is the present value random variable at issue for a 25-year pure endowment of 1 on (x).
- $i = 0.065$
- $\mathrm{Var}[Z] = 0.09\,\mathrm{E}[Z]$

Calculate ${}_{25}p_x$.

Question No. 10:

For a 25-year term life insurance on (40) with varying benefits, you are given:

- Death benefits are payable at the end of the year of death.
- The benefit amount is:
  - (i) 1 in the first 10 years of death,
  - (ii) increasing to 2 for the following 5 years,
  - (iii) increasing further to 3 for the following 5 years, and
  - (iv) remaining at 1 until reaching age 65.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$

Calculate the actuarial present value for this insurance.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK