MATH 3630
Actuarial Mathematics I
Class Test 1 - 3:35-4:50 PM
Wednesday, 15 November 2017
Time Allowed: 1 hour and 15 minutes
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: _____ **Student ID:** _____

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught cheating will be subject to university's disciplinary action.**

Question No. 1:

You are given:

- Z is the present value random variable at issue for an 25-year pure endowment of 10 on (x).
- ${}_{25}p_x = 0.60$
- $\mathrm{Var}[Z] = 1.6\,\mathrm{E}[Z]$

Calculate the annual effective interest rate.

Question No. 2:

Based on the same mortality and interest assumptions, you are given:

- $i = 0.04$
- $\ddot{a}_{65}^{(2)} = 7.7266$ using the Woolhouse's approximation with three terms.
- $\ddot{a}_{65}^{(12)} = 7.5165$ using the Woolhouse's approximation with three terms.

Calculate $\ddot{a}_{65}^{(4)}$ using the Woolhouse's approximation with three terms.

Question No. 3:

Ron is age 65 and just newly retired. He has a total personal savings of 150,000.

He wants guaranteed income while alive. In exchange for a single payment of 150,000, an insurance company promised him an annual payment (at the beginning of each year) of 13,300 with:

- the first 10 payments guaranteed, whether he is alive or not, and
- the subsequent payments made provided he is alive.

You are given:

- $i = 0.05$
- $\ddot{a}_{65} = 10.263$
- $\ddot{a}_{75} = 7.448$

Calculate $\ddot{a}_{65:\overline{10}|}$

Question No. 4:

For a group of 200 lives, each age 65, with independent future lifetimes, you are given:

- Each life is to be paid 2 per month at the beginning of each month, if alive.
- To fund these payments, each life will contribute an amount of c to a fund to support these payments. This contribution is to be made immediately today and only once.
- Y is the present value random variable today of total annuity payments to the 200 lives.
- $i^{(12)} = 0.09$
- $A_{65}^{(12)} = 0.1814$
- ${}^{2}A_{65}^{(12)} = 0.0624$
- The 95^{th} percentile of a standard normal distribution is 1.645.

Using the normal approximation, calculate c such that $\Pr[200c > Y] = 0.95$.

Question No. 5:

For a whole life insurance on (40) with varying benefits, you are given:

- Death benefits are payable at the end of the year of death.
- The benefit amount is:
  - (i) 50 in the first 10 years of death,
  - (ii) increasing to 100 for the following 15 years, but
  - (iii) decreasing to 75 after that until death.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$

Calculate the actuarial present value for this insurance.

Question No. 6:

For a special whole life insurance of 1 issued to (30) with benefits payable at the end of the year of death, you are given:

- Mortality follows the `Illustrative Life Table` except for:
  - ages between 45 and 55 where mortality has a constant force of 0.005.
- $i = 0.06$
- Z is the present value random variable for this insurance.

Calculate $\text{Var}[Z]$.

Question No. 7:

You are given:

- The following select-and-ultimate mortality table with a 3-year select period:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	x+3
55	987	980	973	966	58
56	978	993	966	959	59
57	971	965	958	950	60

- Deaths are uniformly distributed between integral ages.
- $i = 0.05$
- $1000A_{[55]+2.5} = 518$

Calculate $1000_{2.5|}A_{[55]}$.

Question No. 8:

You are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 0.05$
- Y is the present value random variable for a 3-year temporary life annuity-immediate of 1 per year on (65).

Calculate $\text{Var}[Y]$.

Question No. 9:

For the country of Zooturn, you are given:

- Zooturn publishes mortality rates in 3-year intervals, that is mortality rates are of the form: ${}_3q_{3x}$, for $x = 0, 1, 2, \ldots$
- Deaths are assumed to be uniformly distributed between ages $3x$ and $3x + 3$, for $x = 0, 1, 2, \ldots$
- ${}_3q_{48} = 0.02$
- ${}_3q_{51} = 0.05$
- ${}_3q_{54} = 0.09$

Calculate the probability that a person in Zooturn now age 50 will die the next 3.5 years.

Question No. 10:

For a whole life annuity-due of 2 payable at the beginning of each year on (50), you are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 0.05$
- Y is the present value random variable for this annuity.

Calculate the probability that Y exceeds 13.5.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK