**MATH 3630**
**Actuarial Mathematics I**
**Class Test 1 - 3:35-4:50 PM**
**Wednesday, 15 November 2017**
**Time Allowed: 1 hour and 15 minutes**
**Total Marks: 100 points**

**Please write your name and student number at the spaces provided:**

**Name:** ______________________________ **Student ID:** ______________________

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught cheating will be subject to university's disciplinary action.**

## Question No. 1:

You are given:

- The following select-and-ultimate mortality table with a 3-year select period:

| $[x]$ | $\ell_{[x]}$ | $\ell_{[x]+1}$ | $\ell_{[x]+2}$ | $\ell_{x+3}$ | x+3 |
|---|---|---|---|---|---|
| 45 | 1876 | 1871 | 1864 | 1855 | 48 |
| 46 | 1869 | 1864 | 1857 | 1850 | 49 |
| 47 | 1862 | 1856 | 1849 | 1841 | 50 |

- Deaths are uniformly distributed between integral ages.
- $i = 0.06$
- $1000A_{[47]+2.5} = 496$ when $i = 0.06$.

Calculate $1000\,{}_{2.5|}A_{[47]}$.

**Question No. 2:**

For the country of Zoopiter, you are given:

- Zoopiter publishes mortality rates in 5-year intervals, that is mortality rates are of the form: ${}_5q_{5x}$, for $x = 0, 1, 2, \ldots$
- Deaths are assumed to be uniformly distributed between ages $5x$ and $5x + 5$, for $x = 0, 1, 2, \ldots$
- ${}_5p_{40} = 0.97$
- ${}_5p_{45} = 0.94$
- ${}_5p_{50} = 0.90$

Calculate the probability that a person in Zoopiter now age 42 will die the next 6.5 years.

**Question No. 3:**

You are given:

- $Z$ is the present value random variable at issue for an $n$-year pure endowment of 25 on $(x)$.
- ${}_np_x = 0.63$
- $\mathrm{E}[Z] = 3.67$

Calculate $\mathrm{Var}[Z]$.

## Question No. 4:

Becky is age 65 and just newly retired. She has a total personal savings of 1,000,000.

She wants guaranteed income while alive. In exchange for a single payment of 1,000,000, an insurance company promised her an annual payment (at the beginning of each year) of $B$ with:

- the first 10 payments guaranteed, whether she is alive or not, and
- the subsequent payments made provided she is alive.

You are given:

- $i = 0.05$
- $\ddot{a}_{65} = 10.263$
- $\ddot{a}_{75} = 7.448$
- $\ddot{a}_{65:\overline{10}|} = 7.095$

Calculate $B$.

## Question No. 5:

For a whole life insurance on (40) with varying benefits, you are given:

- Death benefits are payable at the end of the year of death.
- The benefit amount is:
  - (i) 100 in the first 5 years of death,
  - (ii) decreasing to 50 for the following 10 years,
  - (iii) decreasing further to 10 for the following 10 years, and
  - (iv) decreasing even further to 5 after that until death.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$

Calculate the actuarial present value for this insurance.

## Question No. 6:

For a special whole life insurance of 1 issued to (30) with benefits payable at the end of the year of death, you are given:

- Mortality follows the `Illustrative Life Table` except for:
  - ages 65 and beyond where mortality has a constant force of 0.01.
- $i = 0.06$
- $Z$ is the present value random variable for this insurance.

Calculate $\mathrm{Var}[Z]$.

### Question No. 7:

For a group of 100 lives, each age 65, with independent future lifetimes, you are given:

- Each life is to be paid 1 per month at the beginning of each month, if alive.
- To fund these payments, each life will contribute an amount of $c$ to a fund to support these payments. This contribution is to be made immediately today and only once.
- $Y$ is the present value random variable today of total annuity payments to the 100 lives.
- $i^{(12)} = 0.06$
- $A_{65}^{(12)} = 0.2965$
- ${}^{2}A_{65}^{(12)} = 0.1191$
- The $95^{\text{th}}$ percentile of a standard normal distribution is 1.645.

Using the normal approximation, calculate $c$ such that $\Pr[100c > Y] = 0.95$.

## Question No. 8:

You are given:

- For a fixed age $x$, ${}_k p_x = (0.92)^k$ for $k \geq 0$.
- $i = 0.05$
- $Y$ is the present value random variable for a 3-year temporary life annuity-immediate of 1 per year on $(x)$.

Calculate $\mathrm{Var}[Y]$.

### Question No. 9:

For a whole life annuity-due of 5 payable at the beginning of each year on $(x)$, you are given:

- Mortality follows a constant force of $\mu = 0.05$.
- $i = 0.035$
- $Y$ is the present value random variable for this annuity.

Calculate the probability that $Y$ exceeds 60.

**Question No. 10:**

Based on the same mortality and interest assumptions, you are given:

- $i = 0.055$
- $\ddot{a}_{50}^{(4)} = 11.3470$ using the Woolhouse's approximation with three terms.
- $\ddot{a}_{50}^{(12)} = 11.2633$ using the Woolhouse's approximation with three terms.

Calculate $\ddot{a}_{50}^{(6)}$ using the Woolhouse's approximation with two terms.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK