MATH 3630
Actuarial Mathematics I
Class Test 2 - 3:35-5:25 PM
Monday, 18 November 2019
Time Allowed: 110 minutes
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: _____ **Student ID:** _____

- **There are twelve (12) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted. Work with not enough details will get reduced marks even if final answer is correct.**
- **Please write legibly.**
- **Anyone caught cheating will be subject to university's disciplinary action.**

Question No. 1:

You are given:

- The following extract from a select-and-ultimate life table with a 2-year select period:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	$x+2$
65	100,000	99,618	97,785	67
66	98,154	97,243	95,816	68
67	96,217	95,219	93,740	69

- Deaths are uniformly distributed between integer ages.

Calculate ${}_{0.90}q_{[65]+0.75}$.

Question No. 2:

Two life insurance policies are actuarially equivalent if they have equal actuarial present values. The following policies issued to (45) are actuarially equivalent:

- A whole life insurance of 250 payable at the end of the year of death.
- A special whole life insurance, also payable at the end of the year of death, that pays 100 for the first 10 years and B thereafter.

You are given:

- $i = 0.05$
- $A_{45} = 0.30$
- $A_{55} = 0.35$
- $A_{45:\overline{10}|} = 0.69$

Calculate the value of B.

Question No. 3:

For a special whole life insurance of 2 issued to (40) with benefits payable at the end of the year of death, you are given:

- Z is the present value random variable for this insurance.
- Mortality follows the `Survival Ultimate Life Table` except for ages:
  - between 50 and 55 where mortality follows a deMoivre's law with $\omega = 100$.
- $i = 0.05$

Calculate $\text{Var}(Z)$.

Question No. 4:

A life insurer has a portfolio of insurance policies consisting of 65% male and 35% female, all of the same age (x) at issue. The forces of mortality for males and females, respectively, are given by

$$\mu^{\mathrm{m}}_{x+t} = 0.048, \quad t > 0$$

and

$$\mu^{\mathrm{f}}_{x+t} = 0.015, \quad t > 0.$$

Let Z be the present value random variable for a whole life insurance of 1 to (x) payable at the moment of death. You are given: $\delta = 0.04$.

For a randomly selected policyholder from this portfolio, calculate the probability that Z will be less than (or equal to) 0.50.

Question No. 5:

You are given:

- For age prior to 50, mortality follows a constant force with $\mu = 0.01$.
- For ages 50 and later, mortality is uniformly distributed with $\omega = 115$.
- $\delta = 0.05$
- Z is the present value random variable for a whole life insurance of 1 issued to (40), with benefit payable at the end of the year of death.

Calculate $\Pr[Z \leq 0.65]$.

Question No. 6:

You and a friend are studying together for Math 3630 midterm exam. One of the practice problems asked to compute the value for A_{50} based on $i = 0.05$.

Your friend calculated the value and came up with 0.357. But the correct answer turned out to be 0.353. She asked you to review her work and you discovered that she used the following set of mortality assumptions:

$$p_{50} = 0.990 \quad \text{and} \quad p_{51} = 0.980.$$

You realized then that all assumptions in her calculations were correct, except for p_{50}.

What must have been the correct value for p_{50}?

Question No. 7:

A special whole life annuity-immediate is issued to (65) with the following increasing scale of benefit payments:

ages	payments
$[65-75)$	100
$[75-90)$	150
90 and later	200

You are given:

- The benefits are payable annually.
- Mortality follows the `Survival Ultimate Life Table`.
- $i = 0.05$

Calculate the actuarial present value of this life annuity-immediate.

Question No. 8:

For a special type of whole life annuity-due, payable monthly, issued to (30), you are given:

- Annuity payments are 5,000 per year for the first 10 years and 1,000 thereafter.
- Deaths are uniformly distributed over integral ages.
- $i = 0.05 \qquad \alpha(12) = 1.00020 \qquad \beta(12) = 0.46651$
- The following table of actuarial present values:

x	A_x	${}_5E_x$
30	0.112	0.780
35	0.139	0.779
40	0.172	0.777

Calculate the Actuarial Present Value (APV) of this life annuity-due.

Question No. 9:

You are given:

- Y is the present value random variable of a (discrete) 2-year deferred 2-year temporary life annuity-due of 1 per year to (95).
- The following extract from a mortality table:

x	94	95	96	97	98	99	100
ℓ_x	100	85	65	40	15	5	0

- $i = 0.05$

Calculate the standard deviation of Y.

Question No. 10:

Based on the same mortality and interest assumptions, you are given:

- $\ddot{a}_{65}^{(4)} = 3.61622$ using the Woolhouse's approximation with three terms.
- $\ddot{a}_{65}^{(6)} = 3.57392$ using the Woolhouse's approximation with three terms.

Calculate $\ddot{a}_{65}^{(12)}$ using the Woolhouse's approximation with three terms.

Question No. 11:

For a group of m lives age x with independent future lifetimes, you are given:

- Each life is to be paid 1 at the beginning of each year, if alive.
- $i = 0.05$
- $\ddot{a}_x = 9.62$
- ${}^2A_x = 0.32$
- Y is the present value random variable of the aggregate payments.
- Using the normal approximation, $F = 2,550$ is the initial size of the fund needed to be 90% certain of being able to make the aggregated payments.
- The 90-th percentile of a standard normal distribution is 1.282.

Calculate m.

Question No. 12:

Consider a special whole insurance policy issued to (45). You are given:

- Death benefit is payable at the end of the year of death.
- The benefit if 10 if death occurs in the first 10 years, 25 if death occurs the following 20 years, and back to 10 for deaths thereafter.
- Level annual premiums P are paid at the beginning of each year for 10 years and decreasing to $0.5P$ per year thereafter.
- Mortality follows the `Survival Ultimate Life Table` with $i = 0.05$.

Calculate P according to the actuarial equivalence principle.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

Standard Ultimate Life Table: Basic Functions and Single Net Premiums at $i = 0.05$

x	l_x	q_x	$\ddot{a}_x$	A_x	^2A_x	$\ddot{a}_{x:\overline{10}\rceil}$	$A_{x:\overline{10}\rceil}$	$\ddot{a}_{x:\overline{20}\rceil}$	$A_{x:\overline{20}\rceil}$	$_5E_x$	$_{10}E_x$	$_{20}E_x$	x
20	100,000.0	0.000250	19.9664	0.04922	0.00580	8.0991	0.61433	13.0559	0.37829	0.78252	0.61224	0.37440	20
21	99,975.0	0.000253	19.9197	0.05144	0.00614	8.0990	0.61433	13.0551	0.37833	0.78250	0.61220	0.37429	21
22	99,949.7	0.000257	19.8707	0.05378	0.00652	8.0988	0.61434	13.0541	0.37837	0.78248	0.61215	0.37417	22
23	99,924.0	0.000262	19.8193	0.05622	0.00694	8.0986	0.61435	13.0531	0.37842	0.78245	0.61210	0.37404	23
24	99,897.8	0.000267	19.7655	0.05879	0.00739	8.0983	0.61437	13.0519	0.37848	0.78243	0.61205	0.37390	24
25	99,871.1	0.000273	19.7090	0.06147	0.00788	8.0981	0.61438	13.0506	0.37854	0.78240	0.61198	0.37373	25
26	99,843.8	0.000280	19.6499	0.06429	0.00841	8.0978	0.61439	13.0491	0.37862	0.78236	0.61191	0.37354	26
27	99,815.9	0.000287	19.5878	0.06725	0.00900	8.0974	0.61441	13.0474	0.37869	0.78233	0.61183	0.37334	27
28	99,787.2	0.000296	19.5228	0.07034	0.00964	8.0970	0.61443	13.0455	0.37878	0.78229	0.61174	0.37310	28
29	99,757.7	0.000305	19.4547	0.07359	0.01033	8.0966	0.61445	13.0434	0.37888	0.78224	0.61163	0.37284	29
30	99,727.3	0.000315	19.3834	0.07698	0.01109	8.0961	0.61447	13.0410	0.37900	0.78219	0.61152	0.37254	30
31	99,695.8	0.000327	19.3086	0.08054	0.01192	8.0956	0.61450	13.0384	0.37913	0.78213	0.61139	0.37221	31
32	99,663.2	0.000341	19.2303	0.08427	0.01281	8.0949	0.61453	13.0354	0.37927	0.78206	0.61124	0.37183	32
33	99,629.3	0.000356	19.1484	0.08817	0.01379	8.0943	0.61456	13.0320	0.37943	0.78199	0.61108	0.37141	33
34	99,593.8	0.000372	19.0626	0.09226	0.01486	8.0935	0.61460	13.0282	0.37961	0.78190	0.61090	0.37094	34
35	99,556.7	0.000391	18.9728	0.09653	0.01601	8.0926	0.61464	13.0240	0.37981	0.78181	0.61069	0.37041	35
36	99,517.8	0.000412	18.8788	0.10101	0.01727	8.0916	0.61468	13.0192	0.38004	0.78170	0.61046	0.36982	36
37	99,476.7	0.000436	18.7805	0.10569	0.01863	8.0905	0.61474	13.0138	0.38029	0.78158	0.61020	0.36915	37
38	99,433.3	0.000463	18.6777	0.11059	0.02012	8.0893	0.61480	13.0078	0.38058	0.78145	0.60990	0.36841	38
39	99,387.3	0.000493	18.5701	0.11571	0.02173	8.0879	0.61486	13.0011	0.38090	0.78130	0.60957	0.36757	39
40	99,338.3	0.000527	18.4578	0.12106	0.02347	8.0863	0.61494	12.9935	0.38126	0.78113	0.60920	0.36663	40
41	99,285.9	0.000565	18.3403	0.12665	0.02536	8.0846	0.61502	12.9850	0.38167	0.78094	0.60879	0.36558	41
42	99,229.8	0.000608	18.2176	0.13249	0.02741	8.0826	0.61511	12.9754	0.38212	0.78072	0.60832	0.36440	42
43	99,169.4	0.000656	18.0895	0.13859	0.02963	8.0804	0.61522	12.9647	0.38263	0.78048	0.60780	0.36307	43
44	99,104.3	0.000710	17.9558	0.14496	0.03203	8.0779	0.61534	12.9526	0.38321	0.78021	0.60721	0.36159	44
45	99,033.9	0.000771	17.8162	0.15161	0.03463	8.0751	0.61547	12.9391	0.38385	0.77991	0.60655	0.35994	45
46	98,957.6	0.000839	17.6706	0.15854	0.03744	8.0720	0.61562	12.9240	0.38457	0.77956	0.60581	0.35809	46
47	98,874.5	0.000916	17.5189	0.16577	0.04047	8.0684	0.61579	12.9070	0.38538	0.77918	0.60498	0.35601	47
48	98,783.9	0.001003	17.3607	0.17330	0.04374	8.0645	0.61598	12.8880	0.38629	0.77875	0.60404	0.35370	48
49	98,684.9	0.001100	17.1960	0.18114	0.04727	8.0600	0.61619	12.8667	0.38730	0.77827	0.60299	0.35112	49
50	98,576.4	0.001209	17.0245	0.18931	0.05108	8.0550	0.61643	12.8428	0.38844	0.77772	0.60182	0.34824	50
51	98,457.2	0.001331	16.8461	0.19780	0.05517	8.0494	0.61670	12.8161	0.38971	0.77711	0.60050	0.34503	51
52	98,326.2	0.001469	16.6606	0.20664	0.05957	8.0431	0.61700	12.7862	0.39113	0.77643	0.59902	0.34146	52
53	98,181.8	0.001623	16.4678	0.21582	0.06430	8.0360	0.61733	12.7527	0.39273	0.77566	0.59736	0.33749	53
54	98,022.4	0.001797	16.2676	0.22535	0.06938	8.0281	0.61771	12.7154	0.39451	0.77479	0.59550	0.33308	54
55	97,846.2	0.001993	16.0599	0.23524	0.07483	8.0192	0.61813	12.6737	0.39649	0.77382	0.59342	0.32819	55
56	97,651.2	0.002212	15.8444	0.24550	0.08067	8.0092	0.61861	12.6271	0.39871	0.77273	0.59109	0.32279	56
57	97,435.2	0.002459	15.6212	0.25613	0.08692	7.9980	0.61914	12.5752	0.40118	0.77151	0.58848	0.31681	57
58	97,195.6	0.002736	15.3901	0.26714	0.09360	7.9854	0.61974	12.5174	0.40393	0.77014	0.58556	0.31024	58
59	96,929.6	0.003048	15.1511	0.27852	0.10073	7.9713	0.62041	12.4531	0.40700	0.76860	0.58229	0.30300	59
60	96,634.1	0.003398	14.9041	0.29028	0.10834	7.9555	0.62116	12.3816	0.41040	0.76687	0.57864	0.29508	60

Standard Ultimate Life Table: Basic Functions and Single Net Premiums at $i = 0.05$

x	l_x	q_x	$\ddot{a}_x$	A_x	^2A_x	$\ddot{a}_{x:\overline{10}\|}$	$A_{x:\overline{10}\|}$	$\ddot{a}_{x:\overline{20}\|}$	$A_{x:\overline{20}\|}$	$_5E_x$	$_{10}E_x$	$_{20}E_x$	x
61	96,305.8	0.003792	14.6491	0.30243	0.11644	7.9379	0.62201	12.3024	0.41417	0.76493	0.57457	0.28641	61
62	95,940.6	0.004234	14.3861	0.31495	0.12506	7.9181	0.62295	12.2145	0.41836	0.76276	0.57003	0.27698	62
63	95,534.4	0.004730	14.1151	0.32785	0.13421	7.8960	0.62400	12.1174	0.42298	0.76033	0.56496	0.26674	63
64	95,082.5	0.005288	13.8363	0.34113	0.14392	7.8712	0.62518	12.0101	0.42809	0.75760	0.55932	0.25569	64
65	94,579.7	0.005915	13.5498	0.35477	0.15420	7.8435	0.62650	11.8920	0.43371	0.75455	0.55305	0.24381	65
66	94,020.3	0.006619	13.2557	0.36878	0.16507	7.8126	0.62797	11.7622	0.43990	0.75114	0.54609	0.23112	66
67	93,398.1	0.007409	12.9542	0.38313	0.17654	7.7781	0.62961	11.6199	0.44667	0.74732	0.53836	0.21764	67
68	92,706.1	0.008297	12.6456	0.39783	0.18862	7.7396	0.63145	11.4643	0.45408	0.74305	0.52981	0.20343	68
69	91,936.9	0.009294	12.3302	0.41285	0.20133	7.6968	0.63349	11.2949	0.46215	0.73828	0.52036	0.18856	69
70	91,082.4	0.010413	12.0083	0.42818	0.21467	7.6491	0.63576	11.1109	0.47091	0.73295	0.50994	0.17313	70
71	90,134.0	0.011670	11.6803	0.44379	0.22864	7.5961	0.63828	10.9118	0.48039	0.72701	0.49848	0.15730	71
72	89,082.1	0.013081	11.3468	0.45968	0.24324	7.5373	0.64108	10.6974	0.49060	0.72039	0.48590	0.14122	72
73	87,916.8	0.014664	11.0081	0.47580	0.25847	7.4721	0.64419	10.4675	0.50155	0.71303	0.47215	0.12511	73
74	86,627.6	0.016440	10.6649	0.49215	0.27433	7.3999	0.64762	10.2221	0.51323	0.70483	0.45715	0.10918	74
75	85,203.5	0.018433	10.3178	0.50868	0.29079	7.3203	0.65142	9.9616	0.52564	0.69574	0.44085	0.09368	75
76	83,632.9	0.020668	9.9674	0.52536	0.30783	7.2325	0.65560	9.6866	0.53873	0.68566	0.42323	0.07887	76
77	81,904.3	0.023175	9.6145	0.54217	0.32544	7.1360	0.66019	9.3980	0.55247	0.67450	0.40427	0.06500	77
78	80,006.2	0.025984	9.2598	0.55906	0.34359	7.0302	0.66523	9.0970	0.56681	0.66217	0.38396	0.05230	78
79	77,927.4	0.029132	8.9042	0.57599	0.36224	6.9146	0.67074	8.7850	0.58166	0.64859	0.36235	0.04096	79
80	75,657.2	0.032658	8.5484	0.59293	0.38134	6.7885	0.67674	8.4639	0.59696	0.63365	0.33952	0.03113	80
81	73,186.3	0.036607	8.1934	0.60984	0.40086	6.6517	0.68325	8.1354	0.61260	0.61727	0.31556	0.02286	81
82	70,507.2	0.041025	7.8401	0.62666	0.42075	6.5037	0.69030	7.8018	0.62848	0.59936	0.29064	0.01616	82
83	67,614.6	0.045968	7.4893	0.64336	0.44094	6.3443	0.69789	7.4651	0.64452	0.57985	0.26498	0.01094	83
84	64,506.5	0.051493	7.1421	0.65990	0.46137	6.1735	0.70602	7.1275	0.66059	0.55868	0.23882	0.00706	84
85	61,184.9	0.057665	6.7993	0.67622	0.48199	5.9915	0.71469	6.7910	0.67662	0.53581	0.21250	0.00431	85
86	57,656.7	0.064554	6.4619	0.69229	0.50272	5.7986	0.72388	6.4574	0.69250	0.51122	0.18635	0.00248	86
87	53,934.7	0.072237	6.1308	0.70806	0.52349	5.5954	0.73355	6.1285	0.70817	0.48492	0.16079	0.00133	87
88	50,038.6	0.080798	5.8068	0.72349	0.54422	5.3828	0.74368	5.8057	0.72354	0.45697	0.13621	0.00066	88
89	45,995.6	0.090326	5.4908	0.73853	0.56484	5.1620	0.75419	5.4903	0.73856	0.42748	0.11305	0.00030	89
90	41,841.1	0.100917	5.1835	0.75317	0.58528	4.9346	0.76502	5.1833	0.75317	0.39659	0.09168	0.00012	90
91	37,618.6	0.112675	4.8858	0.76735	0.60545	4.7021	0.77609	4.8857	0.76735	0.36453	0.07244	0.00005	91
92	33,379.9	0.125708	4.5981	0.78104	0.62529	4.4665	0.78731	4.5981	0.78104	0.33158	0.05559	0.00002	92
93	29,183.8	0.140128	4.3213	0.79423	0.64472	4.2299	0.79858	4.3213	0.79423	0.29808	0.04128	0.00000	93
94	25,094.3	0.156052	4.0556	0.80688	0.66368	3.9945	0.80979	4.0556	0.80688	0.26445	0.02955	0.00000	94
95	21,178.3	0.173599	3.8017	0.81897	0.68209	3.7624	0.82084	3.8017	0.81897	0.23116	0.02029	0.00000	95
96	17,501.8	0.192887	3.5597	0.83049	0.69991	3.5356	0.83164	3.5597	0.83049	0.19872	0.01330	0.00000	96
97	14,125.9	0.214030	3.3300	0.84143	0.71708	3.3159	0.84210	3.3300	0.84143	0.16765	0.00827	0.00000	97
98	11,102.5	0.237134	3.1127	0.85177	0.73356	3.1050	0.85214	3.1127	0.85177	0.13850	0.00485	0.00000	98
99	8,469.7	0.262294	2.9079	0.86153	0.74930	2.9039	0.86172	2.9079	0.86153	0.11173	0.00266	0.00000	99
100	6,248.2	0.289584	2.7156	0.87068	0.76427	2.7137	0.87078	2.7156	0.87068	0.08777	0.00136	0.00000	100