MATH 3630
Actuarial Mathematics I
Class Test 2 - 3:55-5:25 PM
Wednesday, 31 October 2018
Time Allowed: 1.5 hours
Total Marks: 120 points

Please write your name and student number at the spaces provided:

Name: ______________________________ **Student ID:** ______________________

- **There are twelve (12) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**

Question No. 1:

Suppose you are now age 25 and a trust fund worth 1 million has been set up for you. The trust fund has been established so that:

- you receive at the beginning of each year an amount equal to 50,000 while you are alive, and
- your beneficiary receives 1 million at the end of the year of your death.

Assume that your mortality follows the `Survival Ultimate Life Table` and $i = 0.05$.

Calculate the actuarial present value of your trust fund. **Explain**, in words, why this value is higher, equal to, or lower than 1 million.

Question No. 2:

For a whole life insurance of 10 on (45) with death benefit payable at the end of the year of death, let Z be the present value random variables for this insurance. You are given:

- $q_{44} = 0.015$
- $v = 0.962$
- $\dfrac{A_{45}}{A_{44}} = 1.028$
- $\dfrac{{}^2A_{45}}{{}^2A_{44}} = 1.051$

Calculate the standard deviation of Z.

Question No. 3:

For a special life insurance issued to (40), you are given:

- Death benefits are payable at the moment of death.
- The benefit amount is 200 in the in the first 10 years of death, decreasing to 50 after that until reaching age 65.
- An endowment benefit of 500 is paid upon reaching age 65.
- There are no benefits to be paid past the age of 65.
- Mortality follows the `Standard Ultimate Life Table` at $i = 0.05$.
- Deaths are uniformly distributed over each year of age.

Calculate the actuarial present value for this insurance.

Question No. 4:

Mortality is based on the following select and ultimate life table:

x	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	$x+3$
50	5000	4881	4689	4486	53
51	4544	4368	4215	4047	54
52	4096	4009	3889	3731	55

Interest rate is $i = 0.05$.

Calculate $A_{[50]:\overline{5}|}$.

Question No. 5:

Mr. Ow Sum is currently age 40. His mortality follows De Moivre's law with $\omega = 110$.

He buys a temporary life insurance policy that pays him a benefit of \$100 at the moment of his death, if he dies within the next 25 years. No benefits are made if death occurs after 25 years.

You are given that $i = 3.5\%$.

Calculate the actuarial present value of his death benefit.

Question No. 6:

The following is an extract from a life table:

x	ℓ_x
95	15,000
96	12,500
97	9,000

You are given: $\ddot{a}_{95} = 1.785$ and $\ddot{a}_{96} = 1.186$

Calculate i.

Question No. 7:

Each of 150 lives with independent future lifetimes are now age 50 and purchases a whole life insurance of 10 payable at the end of the year of death.

You are given:

- $A_{50} = 0.332$
- ${}^2A_{50} = 0.169$
- Each of the 150 lives pays a one-time premium of c.
- These premiums are calculated so that the probability the insurer has sufficient funds to pay all claims is 0.95.
- The 95th percentile of the standard Normal distribution is 1.645.

Calculate c using the normal approximation.

Question No. 8:

You are given:

- $i = 0.05$
- ${}_{25}q_{40} = 0.3115$
- $\ddot{a}_{40:\overline{25}|} = 1.045 \times a_{40:\overline{25}|}$

Calculate $a_{40:\overline{24}|}$.

Question No. 9:

For a whole life annuity immediate of 100 per year on (67), you are given:

- Mortality follows the `Survival Ultimate Life Table`.
- $i = 0.05$
- Y is the present value random variable for this annuity.

Calculate the probability that Y will exceed 1200.

Question No. 10:

You are given:

- The following is an extract from a life table:

x	ℓ_x	μ_x
60	1000	0.0024
61	994	0.0026
62	988	0.0030
63	978	0.0045

- $i = 0.05$
- Life annuities are approximated using the Woolhouse's formula with three terms.

Calculate $\ddot{a}^{(12)}_{60:\overline{3}|}$.

Question No. 11:

For a whole life annuity-due on (40), you are given:

- Before age 65, mortality follows a constant force $\mu = 0.004$.
- For age 65 and beyond, mortality follows the `Survival Ultimate Life Table`.
- Interest rate $i = 0.10$ for the next 25 years and $i = 0.05$ thereafter.

Calculate $\ddot{a}_{40}$.

Question No. 12:

For a cohort of individuals all age x consisting of non-smokers (ns) and smokers (sm), you are given:

- Mortality is based on the following:

k	q_{x+k}^{ns}	q_{x+k}^{sm}
0	0.01	0.08
1	0.03	0.12

- $i = 0.05$

- $A_{x:\overline{2}|}^{\,1} = 0.0616$ for a randomly chosen individual from this cohort

Determine the proportion of non-smokers and smokers in this cohort at age x.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK