MATH 3630
Actuarial Mathematics I
Class Test 2 - 3:55-5:25 PM
Wednesday, 31 October 2018
Time Allowed: 1.5 hours
Total Marks: 120 points

Please write your name and student number at the spaces provided:

Name: EMIL **Student ID:** Suggested Solutions

- **There are twelve (12) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**

Question No. 1:

Suppose you are now age 25 and a trust fund worth 1 million has been set up for you. The trust fund has been established so that:

- you receive at the beginning of each year an amount equal to 50,000 while you are alive, and
- your beneficiary receives 1 million at the end of the year of your death.

Assume that your mortality follows the `Survival Ultimate Life Table` and $i = 0.05$.

Calculate the actuarial present value of your trust fund. **Explain**, in words, why this value is higher, equal to, or lower than 1 million.

$$APV(\text{trust}) = 50000\, \underbrace{\ddot{a}_{25}}_{19.7090} + 1000000\, \underbrace{A_{25}}_{0.06147}$$

$$= 1{,}046{,}920$$

Interest rate $i = 0.05$ says that 50000 is earned on the trust each year. However, the value of the trust is more than 1 million because you are receiving the interest at the beginning of each year, that you are alive.

If you receive instead an amount equal to $\underbrace{1000000 * \frac{.05}{1.05}}_{47{,}619.05}$,

then the APV will be about 1 million.

Question No. 2:

For a whole life insurance of 10 on (45) with death benefit payable at the end of the year of death, let Z be the present value random variables for this insurance. You are given:

- $q_{44} = 0.015$
- $v = 0.962$
- $\dfrac{A_{45}}{A_{44}} = 1.028$
- $\dfrac{{}^2A_{45}}{{}^2A_{44}} = 1.051$

Calculate the standard deviation of Z.

$$A_{44} = v\,q_{44} + v\,p_{44}\,A_{45}$$

$$\frac{1}{1.028}A_{45} - v\,p_{44}A_{45} = v\,q_{44}$$

$$A_{45}\left(\frac{1}{1.028} - 0.962(.985)\right) = .962(.015)$$

$$\Rightarrow A_{45} = \frac{0.01443}{0.02519265} = 0.5727862$$

$$Var(Z) = \left[{}^2A_{45} - (A_{45})^2\right] * 10^2$$

Similarly,

$${}^2A_{44} = v^2 q_{44} + v^2 p_{44}\,{}^2A_{45}$$

$${}^2A_{45}\left(\frac{1}{1.051} - v^2 p_{44}\right) = v^2 q_{44}$$

$${}^2A_{45} = \frac{0.01388166}{0.03991245} = 0.3478028$$

$$= \left[0.3478028 - (0.5727862)^2\right] * 10^2$$

$$= (0.01971875)\,10^2 \approx \underline{\underline{0.02 * 10^2}} = 1.971875$$

need $SD(Z) =$ ~~0.1404235 ≈ 0.15~~ $\sqrt{1.971875} = 1.404235$

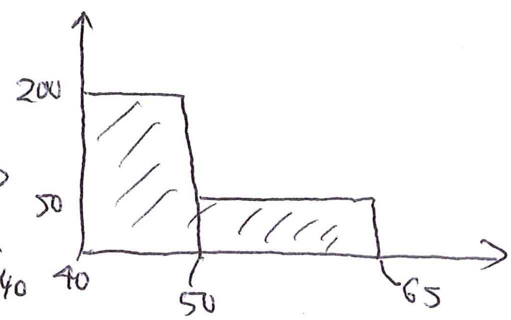
Question No. 3:

For a special life insurance issued to (40), you are given:

- Death benefits are payable at the moment of death.
- The benefit amount is 200 in the in the first 10 years of death, decreasing to 50 after that until reaching age 65.
- An endowment benefit of 500 is paid upon reaching age 65.
- There are no benefits to be paid past the age of 65.
- Mortality follows the `Standard Ultimate Life Table` at $i = 0.05$.
- Deaths are uniformly distributed over each year of age.

Calculate the actuarial present value for this insurance.

$$\text{APV(insurance)} = 200\,\frac{i}{\delta}A_{40} - 150\,\frac{i}{\delta}\,{}_{10}E_{40}\,A_{50} - 50\,\frac{i}{\delta}\,{}_{25}E_{40}\,A_{65} + 500\,{}_{25}E_{40}$$



$i = .05$

$\delta = \log(1.05)$

$A_{40} = 0.12106$

$A_{50} = 0.18931$

$A_{65} = 0.35477$

$${}_{10}E_{40} = v^{10}\frac{l_{50}}{l_{40}} = 0.6092047$$

$${}_{25}E_{40} = v^{25}\frac{l_{65}}{l_{40}} = 0.2811569$$

Plug the values to get

$$\text{APV(insurance)} = \underline{142.55196}$$

Question No. 4:

119 192 203

Mortality is based on the following select and ultimate life table:

x	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	$x+3$
50	5000	4881	4689	4486	53
51	4544	4368	4215	4047	54
52	4096	4009	3889	3731	55

439

316

Interest rate is $i = 0.05$.

Calculate $A_{[50]:\overline{5}|}$.

$$A_{[50]:\overline{5}|} = \frac{1}{5000}\left(119v + 192v^2 + 203v^3 + 439v^4 + 316v^5\right)$$

$$+ v^5 \cdot \frac{3731}{5000}$$

$$= 0.7989878 \approx \underline{\underline{0.80}}$$

Question No. 5:

Mr. Ow Sum is currently age 40. His mortality follows De Moivre's law with $\omega = 110$. He buys a temporary life insurance policy that pays him a benefit of \$100 at the moment of his death, if he dies within the next 25 years. No benefits are made if death occurs after 25 years. You are given that $i = 3.5\%$.

Calculate the actuarial present value of his death benefit.

$\delta = \log(1.035)$

$T_{40} \sim$ Uniform on $(0, 70)$

$f_T(t) = {}_tp_{40}\, \mu_{40+t} = \frac{1}{70}, \quad 0 \le t \le 70$

$$\text{APV(death benefit)} = 100 \int_0^{25} e^{-\delta t} \frac{1}{70}\, dt$$

$$= 100 \times \frac{1}{70} \times \frac{1}{\delta}\left[1 - e^{-\delta(25)}\right]$$

$$= 23.9547 \approx \underline{\underline{23.95}}$$

Question No. 6:

The following is an extract from a life table:

x	ℓ_x
95	15,000
96	12,500
97	9,000

You are given: $\ddot{a}_{95} = 1.785$ and $\ddot{a}_{96} = 1.186$

Calculate i.

$$\ddot{a}_{95} = 1 + v\,p_{95}\,\ddot{a}_{96}$$

$$\underbrace{v}_{\frac{1}{1+i}}\,p_{95} = \frac{\ddot{a}_{95} - 1}{\ddot{a}_{96}}$$

$$i = \frac{\ddot{a}_{96}\,p_{95}}{\ddot{a}_{95} - 1} - 1$$

$$= \frac{1.186\left(\frac{12500}{15000}\right)}{1.785 - 1} - 1$$

$$= 0.259023 \approx \underline{\underline{0.26}}$$

Question No. 7:

Each of 150 lives with independent future lifetimes are now age 50 and purchases a whole life insurance of 10 payable at the end of the year of death.

You are given:

- $A_{50} = 0.332$
- ${}^2A_{50} = 0.169$
- Each of the 150 lives pays a one-time premium of c.
- These premiums are calculated so that the probability the insurer has sufficient funds to pay all claims is 0.95.
- The 95th percentile of the standard Normal distribution is 1.645.

Calculate c using the normal approximation.

$$Z_i = 10v^{K+1}, \quad i = 1, 2, \ldots, 150$$

$$E[Z_i] = 10 A_{50} = 10(0.332) = 3.32$$

$$Var(Z_i) = 100\left({}^2A_{50} - (A_{50})^2\right) = 100\left(.169 - (.332)^2\right) = 5.8776$$

$$Z = Z_1 + \ldots + Z_{150}$$

$$E[Z] = 150(3.32) = 498$$

$$Var(Z) = 150(5.8776) = 881.64$$

$$Pr\left(N \leq \underbrace{\frac{150c - 498}{\sqrt{881.64}}}_{1.645}\right) = 0.95$$

$$150c - 498 = 1.645\sqrt{881.64}$$

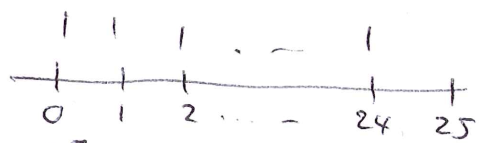
$$c = \frac{498 + 1.645\sqrt{881.64}}{150} = \underline{\underline{3.645627}}$$

Question No. 8:

You are given:

- $i = 0.05$
- ${}_{25}q_{40} = 0.3115$
- $\ddot{a}_{40:\overline{25}|} = 1.045 \times a_{40:\overline{25}|}$

Calculate $a_{40:\overline{24}|}$.



$$\ddot{a}_{40:\overline{25}|} = 1 + a_{40:\overline{25}|} - v^{25}\, {}_{25}p_{40}$$

$$a_{40:\overline{25}|} = \frac{1 - v^{25}\, {}_{25}p_{40}}{.045} = 17.70409$$

$$\ddot{a}_{40:\overline{25}|} = 1.045 \times 17.70409 = 18.50077$$

also equal to $1 + a_{40:\overline{24}|}$

so that $a_{40:\overline{24}|} = \ddot{a}_{40:\overline{25}|} - 1$

$$= 17.50077 \approx \underline{\underline{17.5}}$$

Question No. 9:

For a whole life annuity immediate of 100 per year on (67), you are given:

- Mortality follows the `Survival Ultimate Life Table`.
- $i = 0.05$
- Y is the present value random variable for this annuity.

Calculate the probability that Y will exceed 1200.

$$Y = 100\, a_{\overline{K|}}$$

$$Y > 1200 \Rightarrow 100\,\frac{1-v^K}{i} > 1200$$

$$\Rightarrow v^K < 1-12i$$

$$\Rightarrow K \log v < \log(1-12i)$$

$$\Rightarrow K > \underbrace{\frac{\log(1-12i)}{\log(1/1.05)}}$$

$$\Rightarrow K > 18.78023$$

$$\Pr(K > 18.78023) = \Pr(K \geq 19)$$

$$= 1 - \Pr(K < 19)$$

$$= 1 - \Pr(K \leq 18)$$

$$= 1 - {}_{19}q_{67} = {}_{19}p_{67} = \frac{l_{86}}{l_{67}}$$

$$= \frac{57656.7}{93398.1}$$

$$= \underline{\underline{0.617322}}$$

Question No. 10:

You are given:

- The following is an extract from a life table:

x	ℓ_x	μ_x
60	1000	0.0024
61	994	0.0026
62	988	0.0030
63	978	0.0045

- $i = 0.05$
- Life annuities are approximated using the Woolhouse's formula with three terms.

Calculate $\ddot{a}^{(12)}_{60:\overline{3|}}$.

$$\ddot{a}_{60:\overline{3|}} = 1 + v\frac{994}{1000} + v^2\frac{988}{1000} = 2.842812$$

$\log(1.05)$

$.0024$

$$\ddot{a}^{(12)}_{60:\overline{3|}} = \ddot{a}_{60:\overline{3|}} - \frac{11}{24}\left(1 - {}_3E_{60}\right) - \frac{143}{1728}\left[\delta + \mu_{60} - {}_3E_{60}\left(\delta + \mu_{63}\right)\right]$$

2.842812

$$v^3\frac{l_{63}}{l_{60}} = .8448332$$

$.0045$

$$= \underline{\underline{2.771183}}$$

Question No. 11:

For a whole life annuity-due on (40), you are given:

- Before age 65, mortality follows a constant force $\mu = 0.004$.
- For age 65 and beyond, mortality follows the `Survival Ultimate Life Table`.
- Interest rate $i = 0.10$ for the next 25 years and $i = 0.05$ thereafter.

Calculate $\ddot{a}_{40}$.

$$\ddot{a}_{40} = \underbrace{\ddot{a}_{40:\overline{25}|}}_{\sum_{k=0}^{24} v^k \, {}_kp_{40}} + \underbrace{{}_{25}E_{40}}_{v^{25}\, {}_{25}p_{40}\; \cancel{\frac{l_{65}}{l_{40}}}} \cdot \underbrace{\ddot{a}_{65}}_{13.5498}$$

$v = 1/1.10$, ${}_kp_{40} = e^{-.004k}$, $v = 1/1.10$, ${}_{25}p_{40} = e^{-.004(25)}$

$$= \underbrace{\sum_{k=0}^{24} \left(\frac{e^{-.004}}{1.10}\right)^k} + \left(\frac{1}{1.10}\right)^{25} e^{-.004(25)} \; \cancel{\frac{94579.7}{94338.3}} \times 13.5498$$

$$= \frac{1 - \left(\frac{e^{-.004}}{1.10}\right)^{25}}{1 - \frac{e^{-.004}}{1.10}} + \cdots$$

$$= 9.694359 + \overset{.08351287}{\cancel{.08787475}}\,(13.5498)$$

$$= \cancel{10.88504} \quad \underline{\underline{10.82594}}$$

EV
10/11/2018
Thanks to DS!

Question No. 12:

For a cohort of individuals all age x consisting of non-smokers (ns) and smokers (sm), you are given:

- Mortality is based on the following:

k	q_{x+k}^{ns}	q_{x+k}^{sm}
0	0.01	0.08
1	0.03	0.12

- $i = 0.05$
- $A_{x:\overline{2}|}^{1} = 0.0616$ for a randomly chosen individual from this cohort

Determine the proportion of non-smokers and smokers in this cohort at age x.

$${}^{ns}A_{x:\overline{2}|}^{1} = \frac{1}{1.05}(.01) + \frac{1}{1.05^2}(.99)(.03) = .03646259$$

$${}^{sm}A_{x:\overline{2}|}^{1} = \frac{1}{1.05}(.08) + \frac{1}{1.05^2}(.92)(.12) = .1763265$$

Let ns% be the proportion of non-smokers at age x.

$$A_{x:\overline{2}|}^{1} = ns\% \; {}^{ns}A_{x:\overline{2}|}^{1} + (1-ns\%)\; {}^{sm}A_{x:\overline{2}|}^{1}$$

$$.0616 = ns\% (.03646259) + (1-ns\%)(.1763265)$$

$\Rightarrow$ $ns\% = 82\%$ are non-smokers

so that the rest 18% are smokers!!

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK