MATH 3630
Actuarial Mathematics I
Class Test 2 - 3:35-4:50 PM
Wednesday, 9 November 2016
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL Student ID: Suggested Solutions

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

You are given the following select-and-ultimate mortality table with a 3-year select period:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	x+3
56	875	870	863	856	59
57	868	863	856	849	60
58	861	855	848	840	61

Assume that deaths are uniformly distributed between integer ages.

Calculate ${}_{0.8}q_{[56]+0.5}$.

$$ {}_{0.8}q_{[56]+0.5} = 1 - {}_{0.8}p_{[56]+0.5} $$

$$ = 1 - \frac{\ell_{[56]+1.3}}{\ell_{[56]+0.5}} $$

$$ = 1 - \frac{0.7\,\ell_{[56]+1} + 0.3\,\ell_{[56]+2}}{0.5\,\ell_{[56]} + 0.5\,\ell_{[56]+1}} $$

$$ = 1 - \frac{0.7(870) + 0.3(863)}{0.5(875) + 0.5(870)} $$

$$ = 1 - 0.9947278 $$

$$ = \underline{0.005272206} $$

Question No. 2:

For a 20-year endowment insurance issued to (30), you are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$

Calculate $A_{30:\overline{20|}}$.

$$A_{30:\overline{20|}} = A^{1}_{30:\overline{20|}} + {}_{20}E_{30}$$

$$= A_{30} - {}_{20}E_{30}\, A_{50} + {}_{20}E_{30}$$

$$= 0.10248 - 0.29374(0.24905 - 1)$$

$$= \underline{\underline{0.3230641}}$$

Question No. 3:

For a whole life insurance issued to (x) with benefits payable at the moment of death, you are given:

$$b_t = \begin{cases} 500, & 0 < t \leq 10 \\ 100, & t > 10 \end{cases}$$

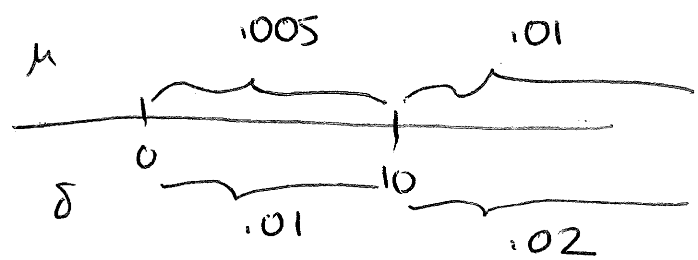
and

$$\delta_t = \begin{cases} 0.01, & 0 < t \leq 10 \\ 0.02, & t > 10 \end{cases}$$

and

$$\mu_{x+t} = \begin{cases} 0.005, & 0 < t \leq 10 \\ 0.010, & t > 10 \end{cases}$$

Calculate the actuarial present value for this insurance.



$$APV = 500 * \frac{.005}{.015}\left(1 - e^{-.015(10)}\right) + 100 * e^{-.015(10)} * \frac{.01}{.03}$$

$$= \underline{\underline{51.9056}}$$

Question No. 4:

For a whole life annuity-due issued to (45), you are given:

- For age before 65, mortality follows exponential distribution with $\mu = 0.01$.
- $\delta = 0.02$
- $A_{65} = 0.91$

Calculate $\ddot{a}_{45}$.

$$A_{45} = \underbrace{A^{\;1}_{45:\overline{20}|}}_{} + \underbrace{{}_{20}E_{45}}_{e^{-.03(20)}} \underbrace{A_{65}}_{0.91}$$

$$\sum_{K=0}^{19} v^{K+1} {}_K p_x \, q_{x+K} = \sum_{K=0}^{19} e^{-.02(K+1)} e^{-.01K} (1-e^{-.01})$$

$$= e^{-.02}(1-e^{-.01}) \underbrace{\sum_{K=0}^{19} e^{-.03K}}_{\frac{1-e^{-.03(20)}}{1-e^{-.03}}}$$

$$= 0.1488947$$

$$A_{45} = 0.1488947 + e^{-.6}(0.91)$$

$$= 0.6483133$$

$$\ddot{a}_{45} = \frac{1-A_{45}}{d} = \frac{1-.6483133}{1-e^{-.02}} = \underline{\underline{17.76077}}$$

Question No. 5:

You are an actuary supervising an actuarial intern. You asked him to verify your calculation for $\ddot{a}_{40}$ based on $i = 5\%$.

You calculated the value and came up with 11.1. But the correct answer, according to the intern, turned out to be 12.1. You asked him to further review your work and he discovered that you used the wrong value of 0.89 for p_{40}.

Calculate the correct value for p_{40}.

use recursion: $\ddot{a}_x = 1 + v p_x \ddot{a}_{x+1}$

actuary: $\ddot{a}_{40} = 1 + v(.89)\ddot{a}_{41} = 11.1$

correct: $\ddot{a}_{40} = 1 + v p_{40} \ddot{a}_{41} = 12.1$

$\ddot{a}_{41} = \dfrac{10.1(1.05)}{.89}$

$p_{40} = \dfrac{11.1(\cancel{1.05})}{10.1(\cancel{1.05})/.89}$

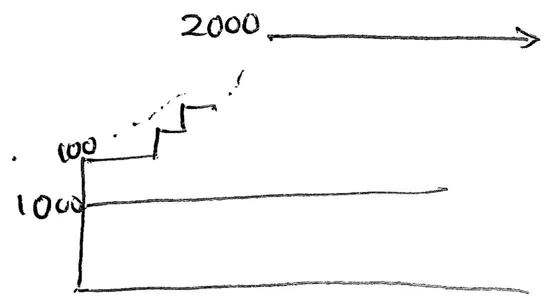
$= \dfrac{11.1}{10.1} 0.89 = \underline{\underline{0.978118}}$

Question No. 6:

should have been 1100

For a special whole life insurance policy issued to (40), you are given:

- Death benefits are payable at the end of the year of death.
- The death benefit in the first year is ~~\$1000~~ 1100, increasing by \$100 each year subsequently until reaching \$2000, after which the benefit remains the same thereafter.
- $A_{40} = 0.42$
- $A_{50} = 0.58$
- $(IA)_{40} = 6.75$
- $(IA)_{50} = 5.95$
- ${}_{10}E_{40} = 0.57$



Calculate the actuarial present value of the death benefits.

$$APV = 1000\, A_{40} + 100\, (IA)_{40} - 100\, {}_{10}E_{40}\, (IA)_{50}$$

$$= 1000(.42) + 100\left[6.75 - 0.57(5.95)\right]$$

$$= \underline{\underline{755.85}}$$

Question No. 7:

You are given:

- Mortality follows de Moivre's law with $\omega = 100$.
- $i = 0.05$

Calculate $\ddot{a}_{45}$.

$$A_{45} = \sum_{K=0}^{54} v^{K+1} \underbrace{{}_K p_{45}}_{\left(1-\frac{K}{55}\right)} \underbrace{q_{45+K}}_{\left(\frac{1}{55-K}\right)}$$

$$= \frac{1}{55} \underbrace{\sum_{K=0}^{54} v^{K+1}}_{\frac{1-v^{55}}{d}} = 0.3387904$$

$$\ddot{a}_{45} = \frac{1-A_{45}}{d} = \frac{1-0.3387904}{.05/1.05} = \underline{\underline{13.8854}}$$

Question No. 8:

For a group of 100 individuals all age x, you are given:

- Their future lifetimes are independent.
- Individuals will be paid a death benefit of \$2 at the moment of their moment.
- $\delta = 0.05$; $\bar{A}_x = 0.40$; and ${}^2\bar{A}_x = 0.20$
- The 95^{th} percentile of a standard normal distribution is 1.645.

Using Normal approximation, calculate the amount of fund needed at inception in order to be 95% certain of having sufficient money to pay the present value of all death benefits.

Let F = amount of fund needed

$\Pr[F \geq S] = 0.95$ where

$E[S] = 100\,\bar{A}_x \times 2 = 40 \times 2 = 80$

$Var[S] = 100 \times 4 \times [0.20 - 0.4^2]$

$= 16$

$\Downarrow$

$\Pr\left[N \leq \dfrac{F-80}{\sqrt{16}}\right] = 0.95$

$\dfrac{F-80}{4} = 1.645 \Rightarrow F = 80 + 4(1.645)$

$= \underline{\underline{86.58}}$

Question No. 9:

You are given:

Mortality follows a select and ultimate life table with a two-year select period:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
50	9706	9687	9661	52
51	9680	9660	9630	53
52	9653	9629	9596	54

$i = 5\%$

Calculate $A^{\ 1}_{[51]:\overline{3}|}$.

$$A^{\ 1}_{[51]:\overline{3}|} = \frac{1}{9680}\left[v(20) + v^2(30) + v^3(34)\right]$$

$$= 0.007812911$$

Question No. 10:

You are given:

- Z is the present value random variable for a whole life insurance of 1 payable at the moment of death of (50).
- Mortality follows de Moivre's law with $\omega = 100$.
- $\delta = 5\%$

Calculate the probability that Z exceeds 0.10.

$$\Pr[Z > 0.10] = \Pr[e^{-\delta T} > 0.10]$$

$$= \underbrace{\Pr\left[T < \frac{\log(0.10)}{-0.05}\right]}$$

$T \sim$ deMoivre's with $\omega - 50 = 50$

$$= \frac{\log(0.10)/-.05}{50} = \underline{\underline{0.921034}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK