MATH 3630
Actuarial Mathematics I
Class Test 2 - 3:35-4:50 PM
Wednesday, 11 November 2015
Time Allowed: 1 hour, 15 minutes
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: ______________________________ **Student ID:** ______________________

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**

Question No. 1:

You are given:

- $A_x = 0.3127$
- $A_{x+20} = 0.5970$
- $A_{x:\overline{20|}} = 0.4225$

Calculate $A^{\,1}_{x:\overline{20|}}$.

Question No. 2:

For a special term life insurance policy issued to (45), you are given:

- Death benefits are payable at the end of the year of death.
- The death benefit in the first year is \$110, increasing by \$10 each year subsequently until reaching \$200, after which the benefit decreases by \$50 each year until reaching zero.
- $A^{\,1}_{45:\overline{10}|} = 0.0957$
- $(IA)^{\,1}_{45:\overline{10}|} = 0.5376$
- $(DA)^{\,1}_{55:\overline{3}|} = 1.8850$
- $(DA)^{\,1}_{55:\overline{4}|} = 2.7400$
- ${}_{10}E_{45} = 0.5354$

Calculate the actuarial present value of the death benefits.

Question No. 3:

For a cohort of individuals all age x consisting of 70% males (m) and 30% females (f), you are given the following extract from a mortality table:

k	q^{m}_{x+k}	q^{f}_{x+k}
0	0.02	0.01
1	0.05	0.03
2	0.08	0.05

Let Z be the present value random variable for a 2-year term life insurance benefit of 1 payable at the end of the year of death of (x). Assume $i = 5\%$.

Calculate $\text{Var}[Z]$ for a randomly chosen individual from this cohort.

Question No. 4:

For a whole life insurance of $10,000 issued to (50), you are given:

- Death benefits are payable at the end of the year of death.
- Mortality follows the `Illustrative Life Table` with the exception of the first year where you are given that $q_{50} = 0.015$.
- The annual effective interest rate is 5% in the first year, 4% in the second year, and 6% each year thereafter.

Calculate the actuarial present value of the death benefits.

Question No. 5:

You are given:

x	q_x	$\ddot{a}_x$
55	0.015	15.84
56	0.020	15.60

Calculate the annual effective interest rate i.

Question No. 6:

For a whole life annuity-due issued to (40), you are given:

- For age before 65, mortality pattern follows the constant force with $\mu = 0.005$.
- $\delta = 0.03$
- $A_{65} = 0.425$

Calculate $\ddot{a}_{40}$.

Question No. 7:

For a special life annuity issued to [55], you are given:

- The benefit payments are \$200 at the beginning of age 57 and \$500 at the beginning of age 59. No other benefit payments are made.
- The following select-and-ultimate mortality table with a 3-year select period:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	x+3
55	882	877	871	864	58
56	875	870	863	856	59
57	868	863	856	849	60
58	861	855	848	840	61
59	854	847	840	832	62
60	846	839	832	823	63

- $i = 0.05$

Calculate actuarial present value of this life annuity.

Question No. 8:

Each of 100 independent lives purchases a single premium whole life insurance of $10 payable at the moment of death. You are given:

- Mortality follows a constant force with $\mu = 0.01$.
- $\delta = 0.05$
- F is the total amount of premium the insurer receives from the 100 lives.
- The 95th percentile of the standard normal distribution is 1.645.

Using a normal approximation, calculate F such that the probability the insurer has sufficient funds to pay all claims is 0.95.

Question No. 9:

You are given:

- Mortality follows de Moivre's law with $\omega = 100$.
- $\delta = 5\%$
- Y is the present value random variable for a whole life annuity-due of \$1 per year issued to (45).

Calculate $\Pr[Y > 12]$.

Question No. 10:

For a whole life insurance on (40) with death benefits payable at the moment of death, you are given:

- The benefit amount at time t is $b_t = 10(1.02)^t$, for $t \geq 0$.
- Mortality follows a constant force with $\mu = 0.015$.
- $\delta = 5\%$

Calculate the actuarial present value for this insurance.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK